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Volatility Clustering and Long Memory-a FIGARCH Analysis of Selected Currency Pairs

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Abstract:

This paper employs the Fractionally Integrated GARCH (FIGARCH) model to analyze the volatility of daily USD returns in four major currency pairs: Euro-USD, GBP-USD, INR-USD, and JPY-USD, using data from January 4, 1999, to August 6, 2021. Before model fitting, the squared-innovation series is examined for the presence of long memory through informal and formal methods. While the Hurst Exponent (H) does not indicate long memory, the Local Whittle Estimator and GPH estimator suggest evidence of long memory for all pairs except JPY-USD.

The findings of the FIGARCH study reveal significant short-term volatility clustering and persistence in volatility, with negligible average returns for the currency pairs. These results contribute to the existing literature on financial time series analysis and have practical implications for market participants. Understanding the presence of long memory in volatility can help traders, risk managers, and policymakers anticipate and manage risks associated with extreme market events and volatility clustering, leading to more informed decision-making in currency markets. The study highlights the importance of considering long memory in volatility when analysing and modelling exchange rate dynamics.

Keywords: FIGARCH, long memory, volatility clustering, exchange rates, risk management.

JEL Classification: C22, C58, F31, G15

1. Introduction:

The complex dynamics of financial time series, particularly in foreign exchange markets, have long fascinated researchers and practitioners alike. Three interrelated and stylized facts about financial time series resulting from autocorrelation-induced dependency are long memory, volatility clusters, and fat tails (Cont, 2001).

Accurately modeling and forecasting the volatility of exchange rates is crucial for effective risk management, asset allocation, and derivative pricing (Andersen et al., 2000).

The seminal works of Engle (1982) and Bollerslev (1986) introduced the ARCH and GARCH models, respectively, which have become the standard tools for capturing volatility clustering in financial time series. However, these models fail to account for the long memory property often observed in the volatility of asset returns (Baillie et al., 1996). Long memory, also known as long-range dependence or persistence, refers to the slow decay of autocorrelations at long lags, implying that past volatility has a long-lasting impact on future volatility (McElroy and Politis, 2020).

To address this shortcoming, Baillie et al. (1996) introduced the Fractionally Integrated GARCH (FIGARCH) model, which extends the GARCH framework by allowing for fractional integration in the volatility process. The FIGARCH model has gained popularity due to its ability to capture both short-term volatility clustering and long memory in financial time series (Davidson, 2004; Belkhouja and Boutahary, 2011).

Despite the growing body of literature on FIGARCH models, the understanding of long memory in the volatility of foreign exchange rates remains limited, particularly for emerging market currencies. Moreover, the practical implications of long memory for market participants, such as traders, risk managers, and policymakers, have not been fully explored.

This paper aims to fill these gaps by employing the FIGARCH model to analyse the volatility of daily USD returns in four major currency pairs: Euro-USD, GBP-USD, INR-USD, and JPY-USD. By examining the presence of long memory in the volatility of these currency pairs, we contribute to the existing literature on financial time series analysis and provide valuable insights for market participants.

Our study is motivated by the need for a better understanding of the long-term dynamics of exchange rate volatility and its potential implications for risk management and decision-making in currency markets. The inclusion of an emerging market currency (INR) alongside major developed market currencies (EUR, GBP, JPY) allows for a comparative analysis and sheds light on the differences in volatility characteristics across markets.

The remainder of this paper is structured as follows. Section 2 reviews the relevant literature on FIGARCH models and their applications in finance. Section 3 describes the data and methodology employed in our analysis. Section 4 presents the empirical results and discusses their implications. Finally, Section 5 concludes the paper and offers suggestions for future research.

2. Literature Review:

The application of fractionally integrated models to capture long memory in financial time series has gained significant attention in the literature. Granger

(1980) introduced the concept of long memory and its implications for economic and financial time series. Hosking (1981) and Geweke and Porter-Hudak (1983) further developed the theory and estimation methods for fractionally integrated processes.

In the context of volatility modeling, Ding et al. (1993) found evidence of long memory in the absolute and squared returns of the SP 500 index. Andersen and Bollerslev (1997) and Baillie et al. (1996) confirmed the presence of long memory in the volatility of foreign exchange rates using high-frequency data and the FIGARCH model, respectively.

Since the introduction of the FIGARCH model by Baillie et al. (1996), numerous studies have applied the model to various financial assets. Bollerslev and Mikkelsen (1996) used the FIGARCH model to analyze the volatility of exchange rates and found evidence of long memory in the Deutsche Mark-US Dollar and Japanese Yen-US Dollar rates. Tse (1998) applied the FIGARCH model to stock market indices and demonstrated its superiority over the GARCH model in capturing long memory.

Beine et al. (2002) investigated the impact of Central bank interventions on the volatility of exchange rates using the FIGARCH model. They found that accounting for long memory and structural breaks improves the fit of the model. Conrad and Haag (2006) examined the persistence of volatility in the FIGARCH model and derived the impulse response function for the conditional variance.

More recently, Kiliç (2011) proposed a smooth transition FIGARCH (ST-FIGARCH) model to capture both long memory and nonlinearity in the volatility of exchange rates and stock market indices. Belkhouja and Boutahary (2011) employed a time-varying FIGARCH model to study the volatility of crude oil prices and found evidence of long memory and structural changes.

In the context of emerging markets, Goudarzi and Ramanarayanan (2010) applied the FIGARCH model to the Indian stock market and found strong evidence of long memory in volatility. Oberholzer and Venter (2015) used the FIGARCH model to analyze the volatility of the South African Rand against major currencies and found that long memory is more pronounced in the Rand-US Dollar exchange rate compared to other currency pairs.

Several studies have compared the performance of FIGARCH with other long memory volatility models. Davidson (2004) introduced the hyperbolic GARCH (HYGARCH) model and showed that it outperforms the FIGARCH model in terms of goodness-of-fit and forecasting ability. Yalama and Sevil (2008) compared the forecasting performance of FIGARCH, HYGARCH, and fractionally integrated APARCH (FIAPARCH) models for the Turkish stock market and found that the FIAPARCH model provides the best out-of-sample forecasts.

3. Research Objectives:

The primary objective of this study is to analyze the volatility of daily USD returns in four major currency pairs: Euro-USD, GBP-USD, INR-USD, and JPY-USD, using the Fractionally Integrated GARCH (FIGARCH) model. Specifically, we aim to:

1. Examine the presence of long memory in the volatility of the selected currency pairs using informal methods, such as the autocorrelation function and periodogram, and formal tests, including the Hurst Exponent, Local Whittle Estimator, and GPH estimator.
2. Estimate the FIGARCH model for each currency pair to capture short-term volatility clustering and long-term persistence in volatility.
3. Assess the significance of the FIGARCH model parameters, particularly the fractional differencing parameter (d), which indicates the degree of long memory in volatility.
4. Investigate the practical implications of the findings for market participants, such as traders, risk managers, and policymakers, in terms of understanding and managing risks associated with extreme market events and volatility clustering.

The relevance of this study lies in its focus on four currency pairs that play a significant role in global financial markets. By examining the presence of long memory in the volatility of these pairs, we contribute to a deeper understanding of how volatility dynamics manifest in these specific markets. The inclusion of an emerging market currency (INR) alongside major developed market currencies (EUR, GBP, JPY) allows for a comparative analysis and provides insights into potential differences in volatility characteristics across markets.

Furthermore, this research aims to bridge the gap between theoretical models and practical applications by discussing the implications of long memory in volatility for market participants. Understanding the persistence of volatility can help investors, risk managers, and policymakers make more informed decisions when dealing with currency risk, portfolio optimisation, and market stability.

Overall, this study contributes to the existing literature on financial time series analysis by providing a comprehensive examination of long memory in the volatility of key currency pairs using the FIGARCH model, while also highlighting the practical significance of the findings for stakeholders in the foreign exchange market.

4. Methodology:

a. The FIGARCH Process:

Let t be an index of time and $P_t, t \in T$ be the observed price of an asset. Let $r_t = \text{In } P_t - \text{In } P_{t-1}$ be its return r_t such that its parameters θ are the maximum likelihood ($L(\theta)$) estimates of an appropriate mean model with pre-chosen density function (f_θ). Then we have:

$$L(\theta; r_1, r_2, ..., r_T) = \prod_{t=1}^{T} \log(f_\theta(r_t)) \tag{4.1}$$

The log-likelihood will be given by:

$$l(\theta; r_1, r_2, ..., r_T) = \sum_{t=1}^{T} \log(f_\theta(r_t)) \tag{4.2}$$

To get the estimator of θ, we maximize $l(\theta)$. Then, the innovations ε_t are:

$$\varepsilon_t = r_t - \theta(B) r_t \tag{4.3}$$

where B is a backshift operator, $\theta(B) = \sum_{i=1}^{n} \theta_i B^i$, and ε_t is white noise with $E(\varepsilon_t) = 0$ and $E(\varepsilon_t^2) = \sigma_t^2$.

The GARCH (p, q) model (Bollerslev, 1986) can be given as:

$$\sigma_t^2 = \omega + \alpha(B)\varepsilon_t^2 + \beta(B)\sigma_t^2 \tag{4.4}$$

where ω, α, $\beta > 0$, and conditional variance is stationary for $\alpha + \beta < 1$.

$\alpha(B) = \sum_{i=1}^{q} \alpha_i B^i$ and $\beta(B) = \sum_{j=1}^{p} \beta_j B^j$. For $q = 0$, it is an ARCH(p) process, and for $p = q = 0$, it becomes a white noise process. After rearranging, we get:

$$(1 - \alpha(B) - \beta(B))\, \varepsilon_t^2 = \omega + (1 - \beta(B))(\sigma_t^2 - \varepsilon_t^2) \tag{4.5}$$

Hence,

$$(1 - B)\, \phi(B)\varepsilon_t^2 = \omega + (1 - \beta(B))(\sigma_t^2 - \varepsilon_t^2) \tag{4.6}$$

Here, $\phi(B) = \sum_{i=1}^{m-1} \phi_i B^i$. In the fractionally integrated model, $(1 - B)$ is replaced by $(1 - B)^d$ for $0 < d < 1$. This is done to capture the slow hyperbolic decay of memory. Thus, we can write the FIGARCH(p, d, q) model equation as:

$$(1 - B)^d\, \phi(B)\, \varepsilon_t^2 = \omega + (1 - \beta(B))(\sigma_t^2 - \varepsilon_t^2) \tag{4.7}$$

We have:

$$(1 - B)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-B)^k \tag{4.8}$$

Solving Equation (7) and truncating the lag polynomial at 1000 for operational purposes (Baillie et al., 1996), we have the FIGARCH(p, d, q) model with $0 < d < 1$ (allowing for slow decline of volatility after volatility shocks, a phenomenon called long memory) as:

$$\sigma_t^2 = (\omega - \bar{\varepsilon}^2) + \sum_{j=1}^{q} \alpha_j(\varepsilon_{t-j}^2\, \bar{\varepsilon}t - j^2) + \sum_{j=1}^{p} \beta_j(\sigma_{t-j}^2 - \varepsilon_{t-j}^2) \tag{4.9}$$

Thus, GARCH(1, 1) and GARCH(1, 1, 1) can be taken as special cases of the FIGARCH(1, d, 1) model with $d = 0$ and $d = 1$, respectively.

4.2 Data and Model Estimation:

The data set consists of daily exchange rates for four currency pairs: Euro-USD, GBP-USD, INR-USD, and JPY-USD. The sample period spans from January 4, 1999, to August 6, 2021, resulting in 5,669 observations for each currency pair.

We first calculate the logarithmic returns for each currency pair as $r_t = \ln P_t - \ln P_{t-1}$, where P_t is the exchange rate at time t. Following Baillie et al. (1996), we fit the mean model $r_t = \mu + \varepsilon_t$ to all four return series.

Before estimating the FIGARCH model, we examine the squared returns for the presence of long memory using informal methods, such as the autocorrelation function (ACF) and periodogram, and formal tests, including the Hurst Exponent (H), Local Whittle Estimator, and GPH estimator.

Finally, we estimate the FIGARCH(1, *d*, 1) model for each currency pair using the maximum likelihood method, assuming normal and Student's *t* distributions for the standardized innovations. The model parameters, including the fractional differencing parameter (*d*), are assessed for statistical significance, and the implications of the findings are discussed.

5. Research Findings:

In this section, we present the results of the FIGARCH analysis of the squared returns of four selected currency pairs: Euro-USD, GBP-USD, JPY-USD, and INR-USD. The FIGARCH model, an extension of the GARCH model, is used to capture volatility and long memory, specifically to model volatility clustering. The model is specified in terms of a univariate conditional mean model, a variance model, a distribution model, and a fractional integration parameter. Before fitting the model to the data, we examine the squared-innovation data for the presence of long memory using the autocorrelation function and periodogram. Additionally, we analyze the data using the Hurst Exponent (H) (Hurst, 1956), Local Whittle Estimator (Robinson, 1995), and GPH estimator (Geweke and Porter-Hudak, 1983).

The ADF test, tau statistics value (with *p*-values indicated in parentheses) for error terms of the four mean models with constant is -15.4397 (1.858e - 036), -17.5335 (1.858e - 036), -27.3391 (1.539e - 051), and -76.5754 (0.0001), respectively. Very low *p*-values indicate that we can reject the null that the four series have unit root. Table 1 shows the summary statistics of the error terms of the mean models of all the four daily return series. All the four series are fat-tailed as indicated by excess kurtosis values. All the popular tests of normality point to the return series being non-normally distributed.

Table 1: Summary Statistics, Returns, 1999-21

Statistics/Currency Pairs	Euro USD	GBP USD	INR USD	JPY USD
Mean	-1.1643e-019	-1.1781e-020	-9.8762e-020	5.1043e-018
Median	-7.6303e-007	8.5727e-005	-9.7861e-005	-0.0012322
Maximum	0.044380	0.044380	0.039285	6.9988
Minimum	-0.081662	-0.081662	- - -0.037658	-0.053389
Std. Dev.	0.0058991	0.0058991	0.0042961	0.093181
Skewness	-0.099739	-0.65764	0.20986	74.763
Ex-Kurtosis	2.4824	10.688	10.810	5612.9
Jarque-Bera Test	1465	27390.6	27643.1	7.44703e+009
P Value	0	0	0	0
Doornik-Hansen Test	808.679	5119.98	6182.23	3.58164e+007
P Value	2.49787e-176	0	0	0
Shapiro-Wilk Test	0.978187	0.946218	0.879669	0.0138291
P Value	1.24335e-028	22.44678e-041	9.7656e-055	6.37173e-099
Lilliefors Test	0.048886	0.0446517	0.109872	0.4295
P Value	0	0	0	0

Source: Author's calculation.

Bai and Ng (2005), with the help of their test specially designed for time series, tested the normality of 21 time series out of these six-time series are found to be non-symmetric and all of them are financial time series. In Table 2, we have presented the result of the Bai & Ng Normality Test for error terms of our mean models for all the four series as most popular tests for normality may not be applicable to financial time series. All return series seem to be non-normally distributed.

Table 2: Bai & Ng Long Run Normality Test, Returns, 1999-21

Currency Pairs	π_3	$\pi_3/\sqrt{5}$	π_4	π_{34}
EuroUSD	1.187239 (0.117567)	2.213913 (0.013417)	3.498400 (0.000234)	19.98223 (4,58E-05)
GBPUSD	0.878083 (0.189949)	0.952704 (0.170370)	1.343738 (0.089517)	3.781008 (0.150996)
INRUSD	0.306107 (0.379762)	0.458222 (0.323397)	3.031943 (0.001215)	10.32863 (0.005717)
JPYUSD	1.440571 (0.074853)	2.335931 (0.009747)	2.854371 (0.002156)	17.10611 (0.000193)

Source: Author's calculation.

Note: p-values are in parentheses. As per Bai and Ng (2005), π_3, $\pi_4 \rightarrow N$, $\pi_{34} \rightarrow x^2$, 5% critical value is 1.96 for π_3, π_4 and 5.99 for π_{34} at 2df.

In Figure 2, we have plotted error square (ε^2) of the mean model of the FIGARCH process of Baillie et al. (1996).

An examination of Figure 2 reveals that all (ε^2) series, except JPY-USD series, have GARCH effect as well as long memory. In the case of Euro-USD, GBP-USD, and INR-USD currency pairs, we have volatility and clustering around the 2008 global financial crisis. For GBP-USD, we have high volatility and clustering around the Brexit period, and in the case of Euro-USD, we also have high volatility clustering during the Brexit period. For autocorrelated series with k lag, we have $\varepsilon^2 \propto \sum_k^T \rho_k$, i.e., clusters in the return series will be caused if lagged observations are significantly correlated, individually or jointly. Individual dependencies can be gauged by autocorrelation function at different lags and joint dependencies can be gauged by portmanteau tests. Long memory is indicated by a slow or hyperbolic decline of autocorrelation as opposed to fast or geometric decline in case of short-term memory. Figure 3 presents ACF of (ε^2) of all the four currency pairs. The autocorrelation function (ACF) at lag k is given by the power rule $\rho_k = c_\rho k^{2d-1}$ with $k \rightarrow \infty$, where c_ρ is a constant and d is a memory parameter. We have a sample size of 5669 for our series, hence 95% confidence interval for autocorrelations, as shown by the horizontal bars in Figure 3, using Bartlett standard error will be given by $\pm \frac{1.96}{\sqrt{5669}} = \pm 0.026$. The volatility clustering phenomenon seen in Figure 1 is confirmed by ACFs given in Figure 2. The (ε^2) autocorrelations for very long lags frequently exceed the two 95% Bartlett (1946) confidence bands except for JPY-USD series. Also, the Ljung and Box (1978) portmanteau test for the joint significance of autocorrelations is quite significant for all lags with p-value equal to zero and this is for all the (ε^2) series except for JPY-USD series for which we do not have any significant Q statistics. This apparent persistence in the autocorrelation function is substantially reduced if we take the first difference of the (ε^2) series. This is indicated by reduced values for ACF though these ACF values are still significant at quite distant lags and the overall significance shown by Q statistics is still very high. The frequency domain definition of long memory states that the spectral density function is unbounded at some frequency in the interval [0,). Most of the empirical literature has concentrated on the case where the singularity, or pole, in the spectrum takes place at the zero frequency (Gil-Alana and Hualde, 2009). We present periodogram, with Bartlett window length of 150, of (ε^2) for all the four series in Figure 4. The figure reveals long memory in the case of all the four return series.

Following Engle and Bollerslev (1986), we will use fractionally differenced return series to capture the long memory in the series. Taylor (2008) studied power transformation of the return from a number of time series and found that squared returns are more likely to be autocorrelated than returns. For a formal examination of long memory, if any, in ε^2, we use Hurst Exponent (H) (Hurst et al., 1965), Local Whittle Estimator (Robinson, 1995), and GPH estimator (Geweke and Porter-Hudak, 1983) of memory parameter d. The results are reported in Table 3 and Table 4, respectively. We do not find evidence of long memory in any of the series on basis of Hurst Exponent (H), but on the basis of Local Whittle Estimator and GPH estimator, we find evidence of long memory. So we don't have a conclusive outcome here (Conclusion).

Following Weron and Weron (2002), we estimate Hurst Exponent (H) = 0.5 + Empirical Hurst Exponent - Anis-Lloyd corrected Hurst Exponent = 0.5 + slope of $(R/S)n$ - $E(R/S)n$. Here $(R/S)n$ is empirical Hurst Exponent estimated by $(R/S)n = CHn$ and $E(R/S)n$ is the expected or theoretical Hurst Exponent at size n (Anis and Lloyd, 1976). There is a lot of debate in literature on the asymptotic property of Hurst Exponent, original R/S analysis gave only the true value of the parameter H but it did not provide its probability distribution. Some researchers have shown that the original Hurst Exponent is normally distributed. Couillard and Davison (2005) studied 10,000 Hurst exponents from independent time series of 10,000 observations each and found them to be normally distributed. In this paper, we use 95% confidence interval suggested by Weron and Weron (2002) for Anis-Lloyd corrected $H(R/S\text{-}AL)$. The estimated value of Empirical Hurst Exponent, Theoretical Hurst Exponent, and Anis-Lloyd corrected $H(R/S\text{-}AL)$ Exponent, along with 95% confidence interval for Anis-Lloyd corrected $H(R/S\text{-}AL)$, is reported in Table 3.

GPH test, $H_0 : d \geq 0.5$, no presence of long memory; $H_1 : d < 0.5$ presence of long memory. LW test, $H_0 : d \geq 0.5$, no presence of long memory; $H_1 : d < 0.5$ presence of long memory, Level of Significance 5%.

Table 3: Hurst Exponent Test for Long Memory in Square Returns

	1. INR USD Sample Size (T) = 5669	**2. Euro USD Sample Size (T) = 5669**	**3. GBP USD Sample Size (T) = 5669**	**4. JPY USD Sample Size (T) = 5669**
Empirical Hurst Exponent $H(R/S)n$	0.7771236	0.8956424	0.8956424	0.8229069
Theoretical Hurst Exponent $H(E(R/S)n)$	0.5287461	0.5287461	0.5287461	0.5287461
Anis-Lloyd Corrected Hurst Exponent $H(R/S-AL)$ $(=0.5+H(R/S)n-H(E(R/S)n))$	0.7483775	0.8668963	0.8604084	0.7908433
95% Confidence Interval for Anis-Lloyd Hurst Exponent	-0.49931 <R/S-AL< 1.408454	-0.49931 <R/S-AL< 1.408454	-0.49931 <R/S-AL< 1.408454	-0.49931 <R/S-AL< 1.408454
H_0	Not rejected	Not rejected	Not rejected	Not rejected
Conclusion	No Long memory	No Long memory	No Long memory	No Long memory

Source: Author's calculation.

Table 4: Frequency Domain Long Memory Estimators

	1. INR USD Sample Size (T) = 5669	**2. Euro USD Sample Size (T) = 5669**	**3. GBP USD Sample Size (T) = 5669**	**4. JPY USD Sample Size (T) = 5669**
Local Whittle Estimator (Robinson, 1995)				
m	177	177	177	177
d	0.291183	0.452571	0.298797	0.344853
Standard Error	0.0375823	0.0375823	0.0375823	0.0375823
Test statistic: $z =$	7.74786	12.0421	7.95048	9.17595
p-value	0.0000	0.0000	0.0000	0.0000
Conclusion	Reject the null. Long memory	Reject the null. Long memory	Reject the null. Long memory	Reject the null. Long memory
GPH Estimator				
m	177	177	177	177
d	0.293947	0.518973	0.297206	0.40157
Standard Error	0.0465883	0.0502571	0.0393567	0.0489204
Test statistic: $t(175) =$	6.30947	0.0502571	7.55159	8.20865
p-value	0.0000	0.0000	0.0000	0.0000
Conclusion	Reject the null. Long memory	Reject the null. Long memory	Reject the null. Long memory	Reject the null. Long memory

Source: Author's calculation.

The estimated values of the FIGARCH(1, *d*, 1) model with normal and Student's *t* distributions for the four currency pairs are presented in Table 5 and Table 6, respectively. The mean return (μ) is estimated to be very close to zero for all currency pairs, suggesting negligible average returns. The constant term (ω) in the volatility equation is not statistically significant for any of the pairs, indicating that the models do not require a constant mean or variance component.

However, both past squared shocks (α_1) and long memory (δ) have significant impacts on current volatility for all currency pairs, suggesting the presence of short-term volatility clustering and strong persistence in volatility. The Student's *t* distribution parameter (ν) is also significant for all currency pairs, indicating the presence of heavy tails in the standardized residuals.

Table 5: FIGARCH (1, *d*, 1) Mean Model: ARFIMA (0, 0, 0) Distribution Model: Normal. Optimal Parameters

Currency Pairs	μ	ω	α_1	β_1	δ
Euro USD	-0.000036 (0.57868)	0.000000 (0.37205)	0.051781 (0.00000)	0.937948 (0.00000)	0.887184 (0.00000)
GBP USD	0.000031 (0.641479)	0.000001 (0.007183)	0.242715 (0.000000)	0.654901 (0.000000)	0.446753 (0.000000)
INR USD	0.000016 (0.626094)	0.000000 (0.012456)	0.151041 (0.000000)	0.914590 (0.000000)	0.956082 (0.000000)
JPY USD	0.000016 (0.626094)	0.000000 (0.012456)	0.151041 (0.000000)	0.914590 (0.000000)	0.956082 (0.000000)

Source: Author's calculation.

These findings have several important implications for market participants. The presence of long memory in volatility suggests that extreme market events and volatility clustering are more likely to occur than in a standard GARCH setting, and market participants should be prepared for prolonged periods of high or low volatility. The heavy tails of the standardized residuals also highlight the importance of accounting for extreme events in risk models and decision-making processes.

Table 6: FIGARCH (1, *d*, 1) Mean Model: ARFIMA (0, 0, 0) Distribution Model: Student's *t* distribution. Optimal Parameters

Currency Pairs	μ	ω	α_1	β_1	δ	ν
Euro USD	-0.000030 (0.634173)	0.000000 (0.144420)	0.052874 (0.000005)	0.937948 (0.00000)	0.883961 (0.00000)	8.385051 (0.000000)
GBP USD	0.000055 (0.39247)	0.000000 (0.19537)	0.004340 (0.82997)	0.951721 (0.000000)	0.984915 (0.000000)	8.313940 (0.00000-)
INR USD	-0.000037 (0.080559)	0.000000 (0.934861)	0.110330 (0.000000)	0.888670 (0.000000)	0.956082 (0.000000)	4.250676 (0.00000)
JPY USD	0.000016 (0.626094)	0.000000 (0.012456)	0.151041 (0.000000)	0.914590 (0.000000)	0.956082 (0.000000)	8.310940 (0.00000)

Source: Author's calculation.

6. Summary and Conclusion:

In this study, we employed the Fractionally Integrated GARCH (FIGARCH) model to analyze the volatility of daily USD returns in four major currency pairs: Euro-USD, GBP-USD, INR-USD, and JPY-USD, using data from January 4, 1999, to August 6, 2021. Our primary objectives were to examine the presence of long memory in volatility, estimate the FIGARCH model to capture short-term volatility clustering and long-term persistence, and investigate the practical implications of our findings for market participants.

In summary, our FIGARCH analysis of the daily USD returns for four major currency pairs (Euro-USD, GBP-USD, INR-USD, and JPY-USD) reveals several important findings. The preliminary analysis shows that the return series are stationary but exhibit fat tails and non-normality. The squared returns display volatility clustering and long memory, particularly for the Euro-USD, GBP-USD, and INR-USD pairs, as evidenced by the autocorrelation function and periodogram.

The formal tests for long memory provide mixed results, with the Hurst Exponent not indicating long memory for any of the series, while the Local Whittle Estimator and GPH estimator suggest evidence of long memory for all pairs except JPY-USD. The FIGARCH (1, *d*, 1) model estimates confirm the presence of short-term volatility clustering and strong persistence in volatility for all currency pairs, as indicated by the significant past squared shocks (α_1) and long memory (δ) parameters. The heavy tails of the standardized residuals are also evident from the significant Student's *t* distribution parameter (ν).

Our study contributes to the existing literature on financial time series analysis by providing a comprehensive examination of long memory in the volatility of key currency pairs using the FIGARCH model. The inclusion of an emerging market currency (INR) alongside major developed market currencies (EUR, GBP, JPY) allows for a comparative analysis and provides insights into potential differences in volatility characteristics across markets.

7. Policy Prescription:

The findings of our study have several important implications for policymakers, particularly central banks and financial regulators. Understanding the presence of long memory in the volatility of major currency pairs can help inform policy decisions related to exchange rate stability, financial market regulation, and risk management.

First, central banks should take into account the persistence of volatility when designing and implementing exchange rate policies. The presence of long memory in volatility suggests that extreme market events and prolonged periods of high or low volatility are more likely to occur than in a standard GARCH setting. This implies that central banks may need to be prepared for more frequent and longer-lasting interventions in foreign exchange markets to maintain stability and prevent excessive fluctuations in exchange rates.

Second, financial regulators should consider the implications of long memory in volatility when setting capital requirements and other risk management standards for financial institutions. The heavy tails of the standardized residuals in our FIGARCH model estimates highlight the importance of accounting for extreme events in risk models. Regulators may need to require financial institutions to hold more capital or maintain higher liquidity buffers to cushion against potential losses arising from prolonged periods of high volatility.

Third, policymakers should promote the development and use of more sophisticated risk management tools that take into account the presence of long memory in volatility. This could include encouraging the adoption of long memory volatility models, such as FIGARCH, HYGARCH, or FIAPARCH, by financial institutions and market participants. Policymakers could also support research and education initiatives aimed at improving the understanding and application of these models in practice.

Fourth, given the evidence of long memory in volatility, policymakers should be cautious about relying too heavily on short-term measures of volatility, such as implied volatility indices or short-term historical volatility, when making policy decisions. These measures may not fully capture the long-term persistence of volatility and could lead to suboptimal policy choices. Instead, policymakers should consider using longer-term measures of volatility and incorporating the insights from long memory models when formulating policies.

Finally, our findings highlight the importance of international cooperation and information sharing among policymakers and regulators. The presence of long memory in volatility in major currency pairs suggests that volatility shocks in one market can have long-lasting effects on other markets. This underscores the need for policymakers to work together to monitor and manage risks arising from cross-border financial market linkages.

8. Limitations and Future Research:

While our study provides valuable insights into the presence of long memory in the volatility of major currency pairs and its implications for market participants and policymakers, it is important to acknowledge the limitations of our analysis and identify areas for future research.

One limitation of our study is the mixed results from the formal tests for long memory. While the Local Whittle Estimator and GPH estimator suggested evidence of long memory for all currency pairs except JPY-USD, the Hurst Exponent did not indicate long memory for any of the series. These mixed results suggest that further research may be needed to fully understand the nature of long memory in these currency pairs and to develop more robust tests for detecting long memory in financial time series.

Another limitation is that our analysis focuses on a specific time period (1999-2021) and may not capture potential changes in volatility dynamics over time. Future research could extend our analysis by considering longer time periods or by using rolling window estimation techniques to examine how the degree of long memory in volatility evolves over time. This could provide insights into how changes in economic conditions, financial market structure, or policy regimes affect the persistence of volatility in foreign exchange markets.

Our study also focuses on a limited set of currency pairs (Euro-USD, GBP-USD, INR-USD, and JPY-USD). While these pairs represent a mix of major developed and emerging market currencies, future research could extend the analysis to a broader set of currency pairs to provide a more comprehensive understanding of long memory in foreign exchange market volatility. This could include examining the presence of long memory in the volatility of other major currency pairs, such as USD-CHF or USD-CAD, as well as in the volatility of currency pairs involving other emerging market currencies, such as USD-BRL or USD-ZAR.

Another area for future research is to explore the use of alternative long memory volatility models, such as the Hyperbolic GARCH (HYGARCH) or the Fractionally Integrated Asymmetric Power ARCH (FIAPARCH) models, to assess the robustness of our findings. These models may provide additional insights into the nature of long memory in volatility and its implications for risk management and policy design.

Future research could also investigate the economic factors driving the differences in long memory across currency pairs. This could involve examining how differences in macroeconomic fundamentals, financial market development, or institutional factors affect the persistence of volatility in different currency markets. Understanding the underlying drivers of long memory in volatility could help market participants and policymakers better anticipate and manage risks arising from volatility persistence.

Finally, future research could explore the implications of long memory in volatility for other financial markets, such as commodities or derivatives. Long memory in volatility may have important implications for the pricing and risk management of financial derivatives, such as options or futures contracts, as well as for the hedging strategies used by market participants. Examining the presence and implications of long memory in volatility across different asset classes could provide a more comprehensive understanding of the role of long memory in financial market dynamics.

Appendix: Figures

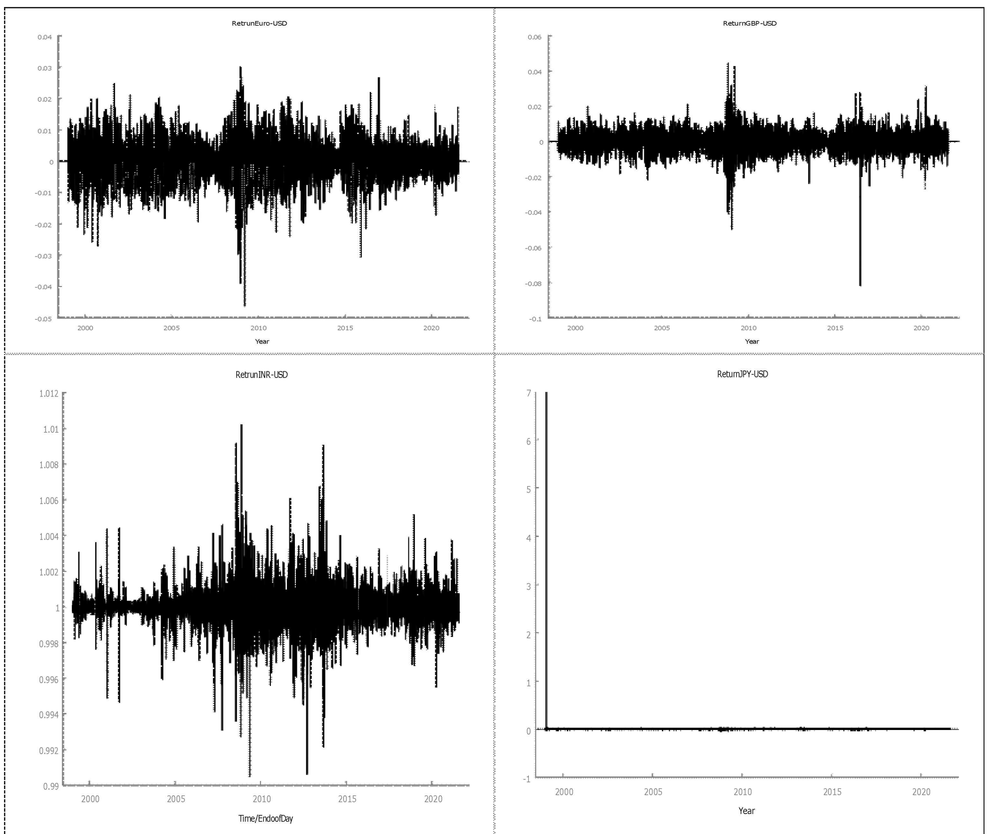

Figure 1: Time Series Plot, Daily Returns Selected Currency Pairs

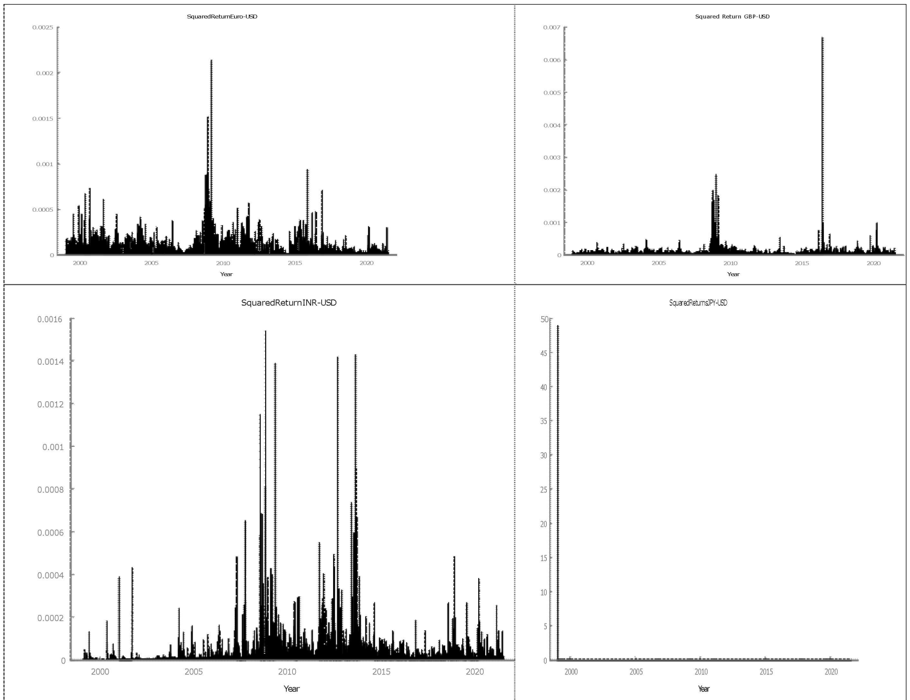

Figure 2: Square of Daily-Rate-of-Return Selected Currency Pairs

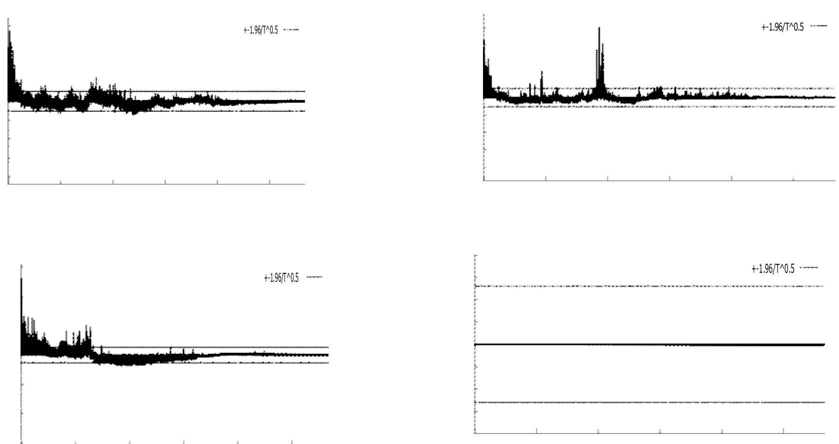

Figure 3: ACF, Squared Returns

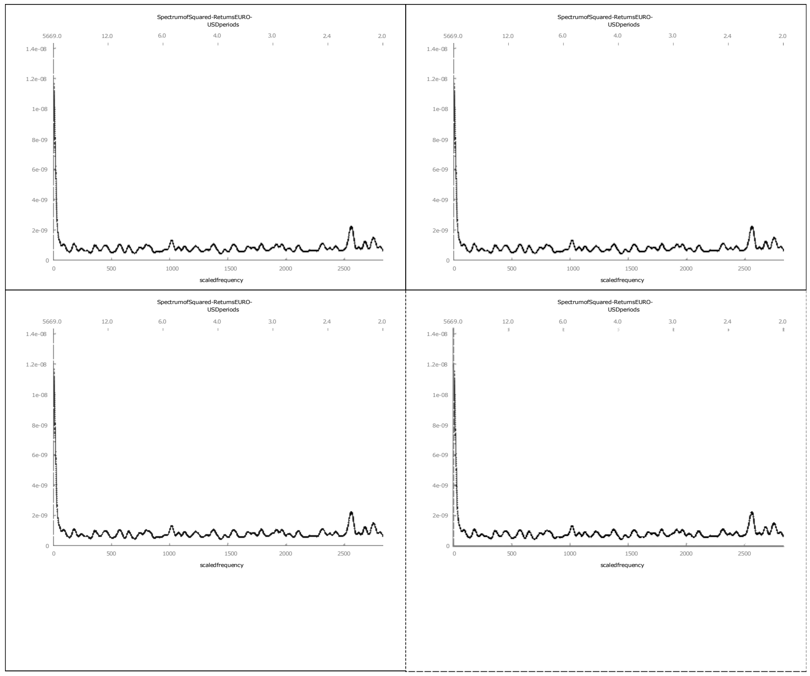

Figure 4: Periodogram, Squared Returns

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