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The Neutrino Survival Probability

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Abstract:

We shall assume the SSM to be correct within its known uncertainties. Given this assumption, the inevitable conclusion is that, somehow, a fraction of the electron neutrinos created within the solar core are lost before they reach the solar neutrino detectors on Earth. The electron neutrino loss is described by the neutrino survival probability, $p(E_\nu)$, which is the probability that an electron neutrino created within the Sun's core arrive at the Earth. Most analyses of the solar neutrino problem posit a particular loss mechanism, based on some assumed new physics; e.g., loss due to the conversion of electron neutrinos to less readity observed states such as muon, tau or sterile neutrinos. They than proceed to determine the model parameters that provide the best fit to data. The degree a fit is good it provides evidence in favour of the particular new physics that has been assumed. Therefore, from this perspective, solar neutrino physics is an avenue for probing physics beyond the standard electroweak theory.

1. Almost all analyses of solar neutrino data make use of X^2 methods to extract information about the model parameters. However, a more powerful way to extract information from data is to use Bayes' theorem (4k8).

$$P(H|D,I) = \frac{P(D|H,I)P(H|I)}{\int_0 P(D|H,I)P(H|I)} \quad \text{(i)}$$

where $P(H|D,I)$ is the probability of hypothesis H, given measured quantities D and prior information I, $P(H|D,I)$ is the probability assigned to data D and $P(H|I)$ is the prior probability assigned to hypothesis H. The integration in the denominator is over all hypothesis of interest. We shall use Bayes' theorem is extract the neutrino survival probability in a manner that provides quantitative information about how well it is known, given current data.

The solar neutrino capture rate S, for the Chlorine and Gallium experiments is given by

$$S_1 = \sum_J \Phi_1 \int_{E\min_t} \sigma_1(E_v)\phi_1(E_v)p(E_v)\,dE_v, \quad \text{(ii)}$$

where Φ_1 is the total flux from neutrino source J, ϕ_1 is the corresponding normalized neutrino energy spectrum, σ_1 is the cross-section for the its experiment, $E_{\min_t}$ is the threshold energy and $p(E_v)$ is the neutrino survival probability.

The Super-Kamiokande experiment (4k1) is sensitive to the high energy neutrinos, in particular, to the ^{8}B neutrino flux. The latter is inferred by measuring the scattering of neutrinos of electrons. In practice Super-Kamiokande measures the electron recoil spectrum. For our analysis, we use the electron recoil spectrum. That spectrum spans the range 6.5 to 20 MeV. Since Super-Kamiokande is sensitive to all neutrino flavors. We must consider two possibilities. It could be that the v, deficit is caused by v conversion to v, where x is either μ or τ. In this case, we must take account of the fact that Super-Kamiokande can record all these neutrino flavors, which different sensitivities and the major flux would then be the sum of these flavors. If, however, the v, are lost through a mechanism that does not result in a detectable signal, e.g. because of conversion into sterile neutrino, then the measured rate would be due to the v, flux only. Both possibilities are considered.

The measured electron recoil spectrum $N(T)$ is given by

$$N(T) = N_0 \int_0^{T^{\max'}(E_\nu^{\max})} dT' R(T \mid T') \times \int_{E_\nu^{\min}(T')}^{E_\nu^{\max}} dE_\nu \, \phi_B(E_\nu)[P(E_\nu)\sigma_e(T', E_\nu) + (1 - P(E_\nu))\sigma_\mu(T', E_\nu)], \quad \text{(iii)}$$

where $R(T|T')$ is the Super-Kamiokande resolution function, which can be approximated by a Gaussian with mean T' and standard deviation $1.5\sqrt{(T'/10\text{MeV})}$, $T = E_\nu - m_s$ and T' are the measured and true electron kinetic energies, respectively, with E_ν and m_s the electron energy and mass. The quantity ϕ_B is the normalized neutrino energy spectrum from the ^{8}B reaction, and σ_B and σ_μ are the ν_s and ν_μ differential electron scattering cross-sections, respectively. For a fixed neutrino energy E_ν, the recoiling electron can have any kinetic energy of

$$T^{\max'}(E_\nu) = 2E_\nu^2/(2E_\nu + m_s), \quad \text{(iv)}$$

while for a fixed electron kinetic energy T' the minimum neutrino energy is given by

$$E_\nu^{\min}(T') = [T' + \sqrt{(T'(T' + 2m_s))}]/2 \quad \text{(v)}$$

We denote by $E_\nu^{\max}$ the maximum possible neutrino energy, which we take to be 20 MeV. The constant N_0 is a normalization factor that depends on the units for the event rate.

Equation (iii) assumes that ν_s are lost through conversion to active neutrinos, e.g., ν_μ. If the measured flux is due to ν_s only, however, the recoil spectrum is given by eq. (iii) with the term proportional to σ_μ omitted. The event rate S_1 in the ith electron energy bin is $N(T)$ integrated over that bin.

Our ability to extract information about the neutrino survival probability depends both on the shape of the solar neutrino spectrum and on the experimental

sensitivity to it. Each experiment is sensitive to different parts of the neutrino energy spectrum as is evident from the (normalized) plots of

$$\sigma(E_\nu)\,\Sigma_1\,\Phi_1\,\phi_1(E_\nu) \qquad \text{(vi)}$$

$$\text{or} \qquad \phi_s(E_\nu) \times \int_{6.5\text{-}m_e}^{20\text{-}m_e} dT \int_0^{T(E_\nu)} dT' R(T\,|\,T')\,\sigma_s(T', E_\nu)$$

for Super-Kamiokande as shown in Fig. 1. We also note the existence of regions where the spectral sensitivity is essentially zero. It should not be expect to learn much about the survival probability in these regions. The probability assigned to the data, i.e. the likelihood function $P(D\,|\,H, I)$ is assumed to be a Gaussian form $g(D\,|\,S, \Sigma)$, where $D \equiv (D_1,..., D_{19})$ represents the 19 data-3 rates from the Chlorine and Gallium experiments plus 16 rates from the binned Super Kamiokande electron recoil spectrum, Σ denotes the 19×19 error matrix for the experimental data and $S = (S_1,..., S_{19})$ represents the predicted rates.

In any Bayesian analysis one must confront the issue of prior probabilities. They are a necessity, indeed, an important part of the Bayesian method. However, this is not the place to get distracted by the intricacies of the prior probability debate. We short circuit discussion by taking, as a matter of convention, the prior probability to be constant. In practice any other plausible choice makes very little difference to our conclusions. Given our desire to be non-committal with respect to models, yet being mindful of previous analysis. We are motivated to write the survival probability as a sum of two finite Fourier series:

$$P(E_\nu|a) = \sum_{r=0}^{7} a_{r+1}\cos(r\pi' E_\nu/L_1)/(1+\exp[(E_\nu - L_1)/b])$$

$$+ \sum_{r=0}^{3} a_{r+2}\cos(r\pi' E_\nu/L_2) \qquad \text{(vii)}$$

The first term in Eq. (vii) is defined in the interval 0.0 to L_1 MeV and suppressed beyond L_1 by the exponential, while the second term covers the interval 0.0 to L_2 MeV. The function is divided this way to accomodate a survival probability that varies rapidly in the interval 0.0 to L_1 and less rapidly elsewhere. Holding the parameters L_1, L_2 and b fixed at 1.0, 15.0 and 0.1 MeV, respectively, we compute the posterior probability $P(a\,|\,D, I)$ for the parameters $a \equiv (a_1,..., a_{12})$. The hypothesis here is that the vector of parameters a assume specified values.

The theoretical uncertainties can be incorporated by treating the fluxes $\Phi \equiv (\Phi_1,.....)$ as parameters with an associated prior probability $P(\Phi|D)$, that encodes the flux predictions and the correlations that exist amongst them. We represent our knowledge of the fluxes by a multivariate Gaussian prior probability $P(\Phi|I) = g(\Phi|\Phi^s, \Sigma_D)$, where $\Phi^2 \equiv (\Phi_1^0,..., \Phi_s^0)$ is the vector of flux predictions and Σ_D is the corresponding error matrix (2).

The posterior probability (using a constant prior probability for the parameters a) is given by

$$P(a, \Phi|D, I) = \frac{P(D|a, \Phi, I)P(\Phi|I)}{\int_a \Phi(D|a, \Phi, I)P(\Phi|I)} \qquad \text{(viii)}$$

which when marginalized (that is, integrated) with respect to Φ gives $P(a|D, I)$. Finally, the survival probability is estimated using

$$P(E_\nu|D, I) = \int_a P(E_\nu|a)P(a|D, I) \qquad \text{(ix)}$$

Our results for the survival probability includes both experimental and theoretical uncertainties. The form obtained agrees with the inferences from previous analyses. But unlike these analyses the one described here provides detailed information about the survival probability and its uncertainties as a function of the neutrino energy. As one might have anticipated, given the sensitivity plot in Figure 1. Our knowledge of the survival probability is very uncertain between 1 and 5 MeV. In fact, the survival probability is slightly constrained in only two narrow regions: in the ^{7}Be (Beryllium-7) region just below 1 MeV and another at around 8 MeV, near the peak of the ^{8}B (Boron-8) neutrino spectrum. For neutrino energies above 12 MeV or so, the survival probability is unconstrained by current data.

The key feature, which is well known, is the marked reduction in the ^{7}Be flux, relative to the prediction of the SSM [2]. This is extremely difficult to explain by any plausible modification of the SSM [2] *k*. Because of its strong temperature dependence, it is easy to reduce the ^{7}Be neutrino flux by slightly reducing the core temperature. But, the temperature dependence of the ^{8}B neutrino flux is considerably stronger, T^{25} compared to T^{10} for ^{7}Be. Therefore, the ^{8}B neutrino flux would be reduced by a much greater factor than that of the ^{7}Be neutrino. Moreover, the ^{8}B

reaction is a daughter reaction of the ^{8}B reaction. Thus the ^{7}Be and ^{8}B neutrino fluxes are reduced, relative to the predictions, in a manner contrary to what one would expect from any plausible lowering of the core temperature. This is the strongest evidence that a non-solar explanation must be sought to explain the loss of electron neutrinos from the Sun.

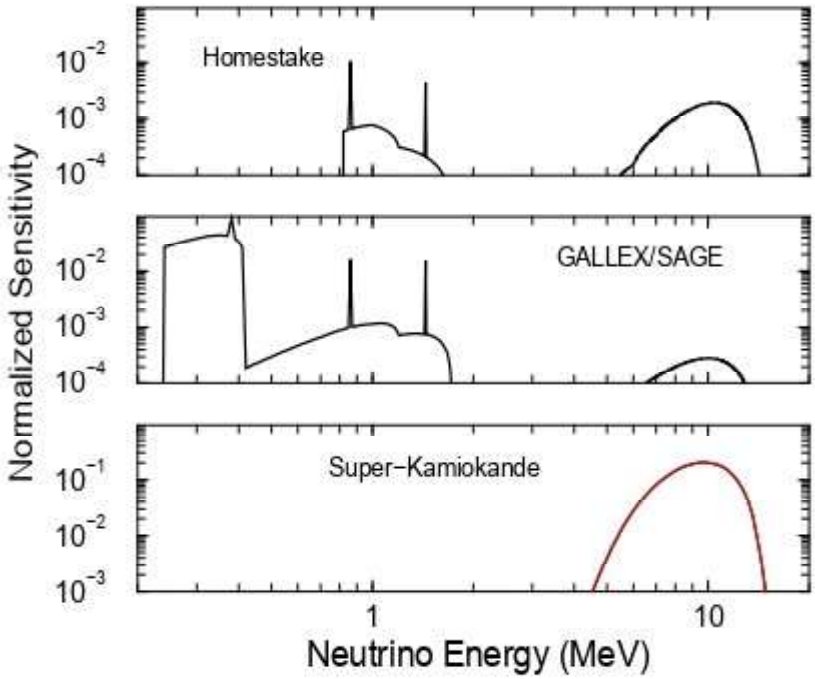


Figure 1: Spectral sensitivity as a function of the neutrino energy

2. Conclusion:

Here we have reviewed the Solar Neutrino Problem and about the Sun and fundamental physics. There is strong evidence that the solar neutrino deficit is real. In particular, the marked suppression of the flux of ^{7}Be neutrinos is very difficult to explain, given the standard solar model. If the SSM is correct, then we are inevitably led to conclude that electron neutrinos are getting iosten route to Earth. What we should infer from this, however, is less clear. Currently, the favoured hypothesis, rendered more probable by the Super-Kamiokande atmospheric neutrino results, invoked some form of neutrino oscillations. In particular, the MSW model provides a good description of the data, which is clearly a strong point in

this favour. But our extraction of the neutrino survival probability suggest that the data could accommodate a wider range of models than have been considered. Therefore, at present, it is not possible to come a definite conclusion regarding the Solar Neutrino Problem.

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