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# Relative Strength of Conic Section

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**Abstract :**

*Conic sections one a group of curves which one generated by slicing a cone with a plane. If the plane is titled parallel to the slope of the cone, the cut produces a parabola. When a parabola is expressed in Cartesian co-ordinates, the second order polynomial. This curve is commonly found in nature, engineering applications and architecture. The study of projectile motion is a real lite application of the parabolic conic section. The projectile moves under the influence of gravity, which for simplicity is assumed to be constant. Thus, it is possible to derive an expression for the height of the projectile as a function of the horizontal position. It turns onto be a second order polynomial that represents a parabola. In the same way, we have obtained different applications of conic section which are more interesting than previous applications.*

**Keywords :** Conic section, projectile, motion, etc.

It is believed that the first definition of a conic section was given by Menaechmus (died 320 BC) as part of his solution of the Delian problem (Duplicating the cube). His work did not survive, not even the names he used for these curves, and is only known through secondary accounts. The definition used at that time differs from the one commonly used today. Cones were constructed by rotating a right triangle about one of its legs so the hypotenuse generates the surface of the cone (such a line is called a generatrix). Three types of cones were determined by their vertex angles (measured by twice the angle formed by the hypotenuse and the leg being rotated about in the right triangle). The conic section was then determined by intersecting one of these cones with a plane drawn perpendicular to a generatrix. The type of the conic is determined by the type of cone, that is, by the angle formed at the vertex of the cone: If the angle is acute then the conic is an ellipse; if the angle is right then the conic is a parabola; and if the angle is obtuse then the conic is a hyperbola (but only one branch of the curve).

Euclid (fl. 300 BC) is said to have written four books on conics but these were lost as well. Archimedes (died c. 212 BC) is known to have studied conics, having determined the area bounded by a parabola and a chord in Quadrature of the Parabola. His main interest was in terms of measuring areas and volumes of figures related to the conics and part of this work survives in his book on the solids of revolution of conics, On Conoids and Spheroids.

**Apollonius of Perga :**

Diagram from Apollonius' *Conics*, in a 9th-century Arabic translation

The greatest progress in the study of conics by the ancient Greeks is due to Apollonius of Perga (died c. 190 BC), whose eight-volume Conic Sections or Conics summarized and greatly extended existing knowledge. Apollonius's study of the properties of these curves made it possible to show that any plane cutting a fixed double cone (two napped), regardless of its angle, will produce a conic according to the earlier definition, leading to the definition commonly used today. Circles, not constructible by the earlier method, are also obtainable in this way. This may account for why Apollonius considered circles a fourth type of conic section, a distinction that is no longer made. Apollonius used the names 'ellipse', 'parabola' and 'hyperbola' for these curves, borrowing the terminology from earlier Pythagorean work on areas.

Pappus of Alexandria (died c. 350 AD) is credited with expounding on the importance of the concept of a conic's focus, and detailing the related concept of a directrix, including the case of the parabola (which is lacking in Apollonius's known works).

**Al-Kuhi :**

An instrument for drawing conic sections was first described in 1000 AD by the Islamic mathematician Al-Kuhi.

**Omar Khayyám :**

Apollonius's work was translated into Arabic, and much of his work only survives through the Arabic version. Persians found applications of the theory, most notably the Persian mathematician and poet Omar Khayyám, who found a geometrical method of solving cubic equations using conic.

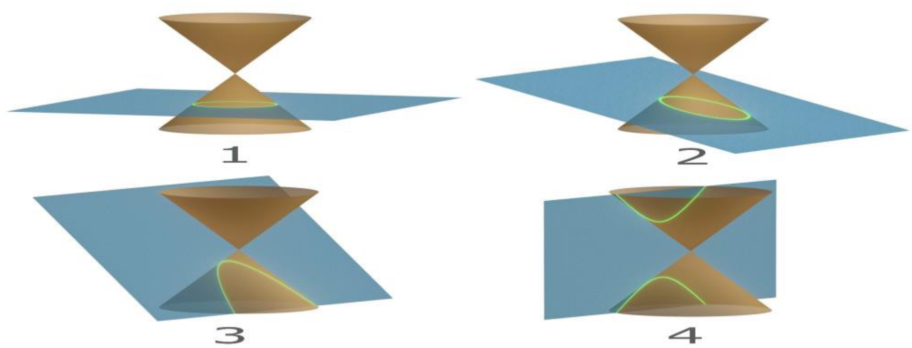
**Europe :**

Johannes Kepler extended the theory of conics through the "principle of continuity", a precursor to the concept of limits. Kepler first used the term 'foci' in 1604. Girard Desargues and Blaise Pascal developed a theory of conics using an early form of projective geometry and this helped to provide impetus for the study of this new field. In particular, Pascal discovered a theorem known as the hexagrammum mysticum from which many other properties of conics can be deduced.

René Descartes and Pierre Fermat both applied their newly discovered analytic geometry to the study of conics. This had the effect of reducing the geometrical problems of conics to problems in algebra. However, it was John Wallis in his 1655 treatise Tractatus de sectionibus conicis who first defined the conic sections as instances of equations of second degree. Written earlier, but published later, Jan de Witt's Elementa Curvarum Linearum starts with Kepler's kinematic construction of the conics and then develops the algebraic equations. This work, which uses Fermat's methodology and Descartes' notation has been described as the first textbook on the subject. De Witt invented the term 'directrix'.

**Definition of Conic Section :**

In mathematics, a conic section (or simply conic) is a curve obtained as the intersection of the surface of a cone with a plane. The three types of conic section are the hyperbola, the parabola, and the ellipse; the circle is a special case of the ellipse, though historically it was sometimes called a fourth type. The ancient Greek mathematicians studied conic sections, culminating around 200 BC with Apollonius of Perga's systematic work on their properties.




The conic sections in the Euclidean plane have various distinguishing properties, many of which can be used as alternative definitions. One such property defines a non-circular conic[1] to be the set of those points whose distances to some particular point, called a focus, and some particular line, called a directrix, are in a fixed ratio, called the eccentricity. The type of conic is determined by the value of the eccentricity. In analytic geometry, a conic may be defined as a plane algebraic curve of degree 2; that is, as the set of points whose coordinates satisfy a quadratic equation in two variables, which may be written in matrix form. This equation allows deducing and expressing algebraically the geometric properties of conic sections.

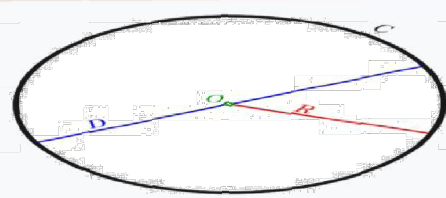
In the Euclidean plane, the three types of conic sections appear quite different, but share many properties. By extending the Euclidean plane to include a line at infinity, obtaining a projective plane, the apparent difference vanishes: the branches of a hyperbola meet in two points at infinity, making it a single closed curve; and the two ends of a parabola meet to make it a closed curve tangent to the line at infinity. Further extension, by expanding the real coordinates to admit complex coordinates, provides the means to see this unification algebraically.

**Types of Conic Section :**

**1) Circle :**

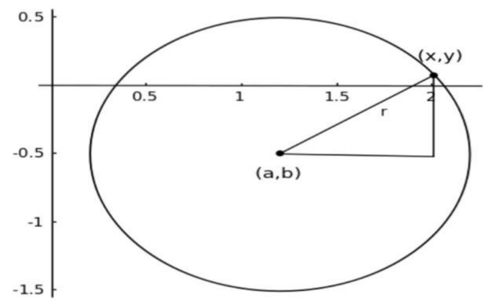
A circle is a shape consisting of all points in a plane that are at a given distance from a given point, the centre. Equivalently, it is the curve traced out by a point that moves in a plane so that its distance from a given point is constant. The distance between any point of the circle and the centre is called the radius. Usually, the radius is required to be a positive number. A circle with $r = 0$ is a degenerate case. This article is about circles in Euclidean geometry, and, in particular, the Euclidean plane, except where otherwise noted.

**Circle**




A circle (black), which is measured by its circumference (*C*), diameter (*D*) in blue, and radius (*R*) in red; its centre (*O*) is in green.

| | |
|---|---|
| Type | Conic section |
| Symmetry group | O(2) |
| Area | $\pi R^2$ |
| Perimeter | $C = 2\pi R$ |




Circle of radius $r$ = 1, centre ($a$, $b$) = (1.2, −0.5)

Specifically, a circle is a simple closed curve that divides the plane into two regions: an interior and an exterior. In everyday use, the term "circle" may be used interchangeably to refer to either the boundary of the figure, or to the whole figure including its interior; in strict technical usage, the circle is only the boundary and the whole figure is called a disc.

A circle may also be defined as a special kind of ellipse in which the two foci are coincident, the eccentricity is 0, and the semi-major and semi minor axes are equal; or the two-dimensional shape enclosing the most area per unit perimeter squared, using calculus of

Equations
Cartesian coordinates
Equation of a circle

In an $x$ - $y$ Cartesian coordinate system, the circle with centre coordinates ($a$, $b$) and radius $r$ is the set of all points ($x$, $y$) such that

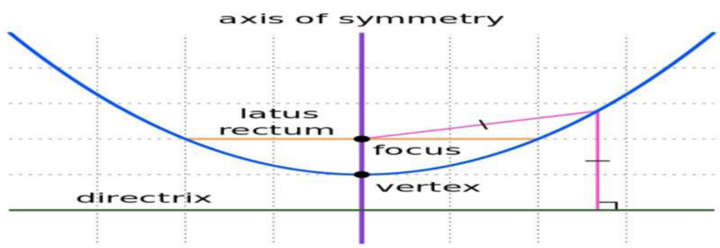
$$(x - a)^2 + (y - b)^2 = r^2$$

This equation, known as the equation of the circle, follows from the Pythagorean theorem applied to any point on the circle: as shown in the adjacent diagram, the radius is the hypotenuse of a right-angled triangle whose other sides are of length $Ix - al$ and $ly - bl$. If the circle is centred at the origin (0, 0), then the equation simplifies to

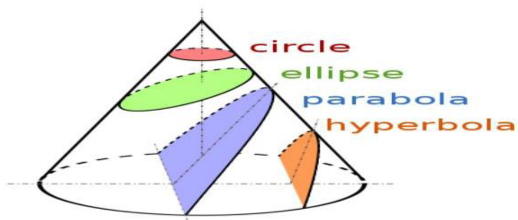
$$x^2 + y^2 = r^2$$

**2) Parabola :**

In mathematics, a parabola is a plane curve which is mirror-symmetrical and is approximately U shaped. It fits several superficially different mathematical descriptions, which can all be proved to define exactly the same curves.




Part of a parabola (blue), with various features (other colours). The complete parabola has no endpoints. In this orientation, it extends infinitely to the left, right, and upward.



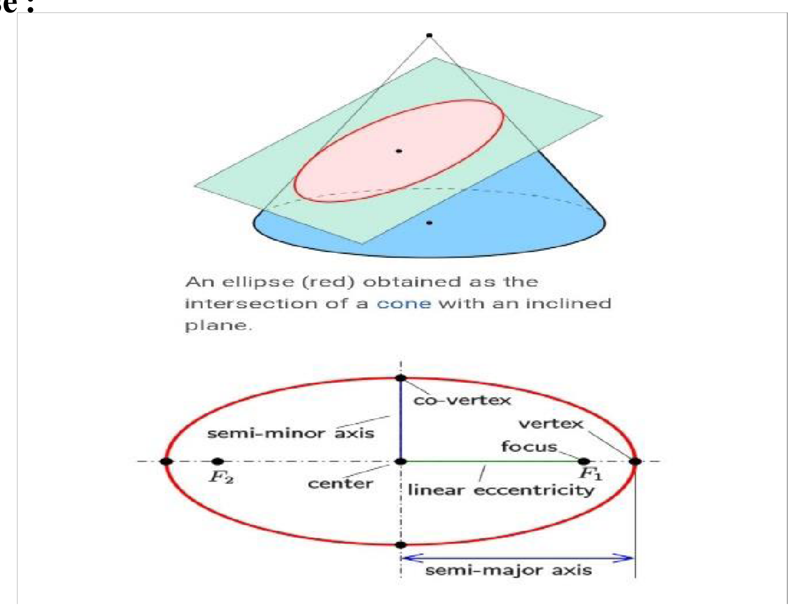

The parabola is a member of the family of conic sections.

One description of a parabola involves a point (the focus) and a line (the directrix). The focus does not lie on the directrix. The parabola is the locus of points in that plane that are equidistant from both the directrix and the focus. Another description of a parabola is as a conic section, created from the intersection of a right circular conical surface and a plane parallel to another plane that is tangential to the conical surface.

The line perpendicular to the directrix and passing through the focus (that is, the line that splits the parabola through the middle) is called the "axis of symmetry". The point where the parabola intersects its axis of symmetry is called the "vertex" and is the point where the parabola is most sharply curved. The distance between the vertex and the focus, measured along the axis of symmetry, is the "focal length". The "latus rectum" is the chord of the parabola that is parallel to the directrix and passes through the focus. Parabolas can open up, down, left, right, or in some other arbitrary direction. Any parabola can be repositioned and rescaled to fit exactly on any other parabola-that is, all parabolas are geometrically similar. Parabolas have the property that, if they are made of material that reflects light, then light that travels parallel to the axis of symmetry of a parabola and strikes its concave side is reflected to its focus, regardless of where on the parabola the reflection occurs. Conversely, light that originates from a point source at the focus is reflected into a parallel ("collimated") beam, leaving the parabola parallel to the axis of symmetry. The same effects occur with sound and other waves. This reflective property is the basis of many practical uses of parabolas.

The parabola has many important applications, from a parabolic antenna or parabolic microphone to automobile headlight reflectors and the design of ballistic missiles. It is frequently used in physics, engineering, and many other areas.

**3) Ellipse :**




An ellipse is the locus of a point in a plane such that the sum of the distances of the point from two fixed points is constant.

In mathematics, an ellipse is a plane curve surrounding two focal points, such that for all points on the curve, the sum of the two distances to the focal points is a constant. As such, it generalizes a circle, which is the special type of ellipse in which the two focal points are the same. The elongation of an ellipse is measured by its eccentricity $e$, a number ranging from $e =: 0$ (the limiting case of a circle) to $e = 1$ (the limiting case of infinite elongation, no longer an ellipse but a parabola).

for an ellipse centered at the origin with its major axis on the $X$-axis and

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

for an ellipse centered at the origin with its major axis on the $Y$-axis.

**4. Hyperbola :**

A hyperbola is the locus of a point in a plane such that the difference of the distances from two fixed points is constant.

If $e > 1$ then the econic is known as Hyperbola.

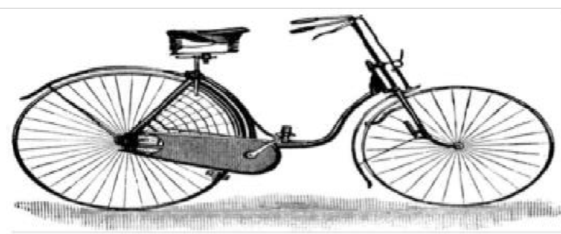
A hyperbola is the locus of a point in a plane such that the differences from the two fixed points in Constant.

**Applications of Conic Sections :**

**Applications of Circle :**

***1. Wheels :***

Circles are the best shape for a bicycle because they roll very easily as they are round. The center point would be the $(h, k)$ in the equation and all points along the outer edge would be the $(x, y)$ values. The radius would be represented by the bars supporting the wheel that run from center to the outer rim.



***Clocks :***

It is a cycle of 60 seconds, 60 min and 12 hours. 60/12 = 5 hence 5 minutes increments, circle because cycles are circular they repeat once the cycle runs through easiest way to repeat the cycle in a circular or loop.

**Applications of Parabola :**

***1. Parabolic receivers :***

A person who whispers at the focus of one of the parabolic reflectors can be heard by a person located near the focus of the other Parabola.

***2. Satellite Dish :***

The shape of a cross section of a satellite antenna is a parabola. The shape of the antenna is a paraboloid. The 'dish' part of the antenna is just a reflective surface. The actual antenna is the object held up in the center by an arm that comes off the side of the dish. The antenna is positioned at the focus. Any energy that is parallel to the axis of the parabola will reflect back to the focus of the parabola regardless of where the energy strikes the surface.

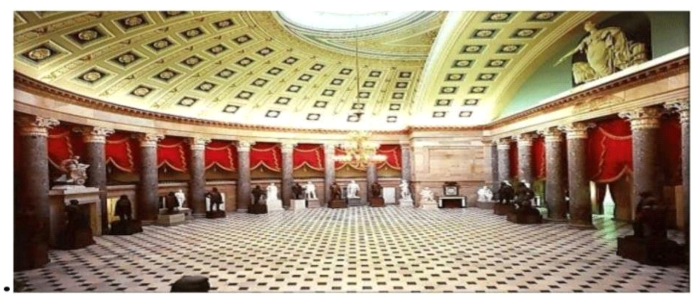
*3. Free flight :*

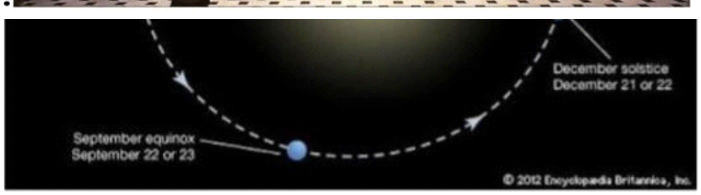
The trajectory of a ball is a free flight. As you throw a ball, it first goes up and forward, then falls down while continuing to travel forward, thus forming an inverted parabola path. Hitting a baseball will cause its path to form a parabola.

**Applications of Ellipse :**

*1. Astronomy :*

Ellipses show up in many areas of astronomy. The ellipse plays an important part in astronomy. Early astronomers believed planets orbited in a perfectly circular pattern, but Johannes Kepler proved that they follow an elliptical orbit and later used the properties of ellipses to create a set of laws about the universe. Using these laws, along with the mathematics of ellipses, astronomers can predict the arrival of comets, planetary orbit and other physical laws.





**NASA :**

Without the ability to actually go to planets and take physical measurements, astrophysicists have to make estimates about their measurements. In 2003, NASA sent remote-controlled land vehicles to Mars. To give the astronauts a visual landing zone they used the shape of an ellipse. The ellipse gave them an area inside of safe terrain and the mathematics of ellipses allowed them to better calculate the chance of landing outside of the predicted zone.

***2. Whispering Gallery :***

An ellipse has the property that if light or a sound wave emanates from one focus, it will be reflected to the other focus. A focus is one of two points that defines the shape and size of the ellipse. This property is used to create whispering galleries, which are structures that allow someone who is whispering in one area to be heard clearly by someone in another area but not by anyone else. Famous examples of whispering galleries include the United States Statuary Capitol Hall and London's St. Paul's Cathedral.

It was in this room that John Quincy Adams, while a member of the House of Representatives, is covered this acoustical phenomenon. He situated his desk at a focal point of the elliptical ceiling easily eavesdropping on the private conversations of other House members located near the other focal point.

***3. Optics :***

Ellipses have important applications to optics. Ellipses show up in nearly all fields of optics. The two foci of an ellipse allow opticians to accurately predict the path of a beam of light. By manipulating the angle and size of an elliptical lens, they can magnify, refract or reflect light. They then use these properties to create microscopes, telescopes and cameras. Physicists and engineers use the optical properties of ellipses to determine how much light scatters and how much an object absorbs - two important properties of Laser mechanics.

**Applications of Hyperbola :**

***1. Lampshade :***

A household lamp casts hyperbolic shadows on a wall

***2. Towers of Nuclear Reactors :***

The hyperboloid is the design standard for all nuclear cooling towers. It is structurally sound and can be built with straight steel beams.

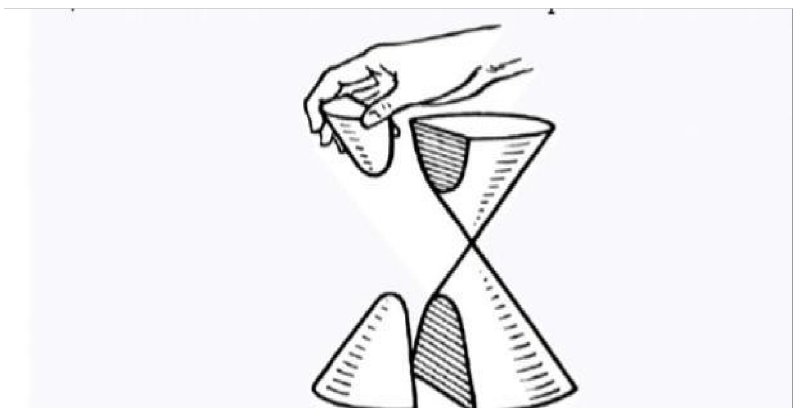
When designing these cooling towers, engineers are faced with two problems:

(1) the structure must be able to withstand high winds and
(2) they should be built with as little material as possible.

The hyperbolic form solves both of these problems. For a given diameter and height of a tower and a given strength, this shape requires less then any other form. A 500 foot tower can be made of a reinforced concrete Shell only six or eight inches wide. See the pictures below (this nuclear power Plant is located in Indiana).

### *3. Trilateration :*

Trilateration is the a method of pinpointing an exact location, using its distances to a given points. The can also be characterized as the difference in arrival times of synchronized signals between the desired point and known points. These type of problems arise in navigation, mainly nautical. A ship can locate its position using the arrival times of signals from GPS transmitters. Alternatively, a homing beacon can be located by comparing the arrival times of its signals at two separate receiving stations. This can be used to track people, cell phones, internet signal and many other things. In particular, the set of possible positions of a point that has a distance variation of $2a$ from two known points is a hyperbola of vertex separation $2a$, and whose foci are the two known points.



### Applications of Conic Sections in our Life :

In our daily life, there are lots of things which uses Conic Sections in Our Surroundings. But we din't even notice that. After getting this project we Noticed and realized it. Conic section are found in different shapes like Circle, Parabola, Ellips and Hyperbola. With the help of Conic Section, different shapes and sizes of things are formed. It gives us the right way to use Conic section In our Life. By using Conic section, the shape that is made at "Ramanand Chowk" of Janakpur is the shape of Parabola. Similarly, "Sahid Dywar" Which is made Mujeliya by using Parabola. In this way, there are lots of benefits and uses of Conic section in our daily life.

**Ramanand Chowk (Asia's Tallest Entrance Gate)**

"Asia's Tallest Entrance Gate" which is made Ramanand Chowk by using Parabola.

**Sahid Guwar**

By using conic section, the shape that is at Mujeliya of Janakpur is the shape of parabola. There are two parabola shape.

**Introduction :**

History of projectile Galileo was the first person who ever accurately described projectile motion. He was the one who projectile motion, first broke down motion into its separate horizontal and vertical components (web) Galileo even took this idea farther with his seat realization that there was more than one force at work upon the Projectile. Utilizing these revolutionary insights, he then concluded that the curve of any projectile is a parabola. It was this idea that let us examine how two objects in a vacuum, even if of different shape and mass, will hit the ground at the exact same time, if both are dropped from the same height.

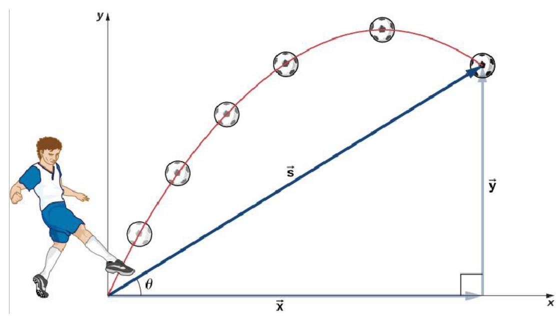
**Definition :**

A projectile is any object thrown into space upon which the only acting force is gravity. The primary force acting on a projectile is gravity.

**Theory :**

**Projectile motion :**

Projectile is defined as, any body thrown with some initial velocity, which is then allowed to move under the action of gravity alone, without being propelled by any engine or fuel. The path followed by *a* is called its trajectory. A projectile moves at a constant speed in the horizontal. While experiencing a constant acceleration of 9.8m/$s^2$ downwards in the vertical direction. To be consistent , we define the up or upwards direction to be the positive direction. Therefore are the acceleration of gravity is, -9.8 m/$s^2$.




**Horizontal motion of Projectile :**

The speed in the horizontal direction is '$Vx$' and this speed doesn't change. The equation which predicts the position at any time in the horizontal direction is simply,

$$X = V \times t$$

**Vertical motion of Projectile :**

Because gravity has a downward pull, the vertical velocity changes constantly. The equation that predicts the vertical velocity at any time $Vy$ is

$$Vy = Vay + at$$

The '*Vay*' is simply the original velocity in the vertical or *y*-direction.

To calculate the position in the *y*-direction, the full distance formula must be used. "$Y_0$" represents the original position in the *y*-direction.

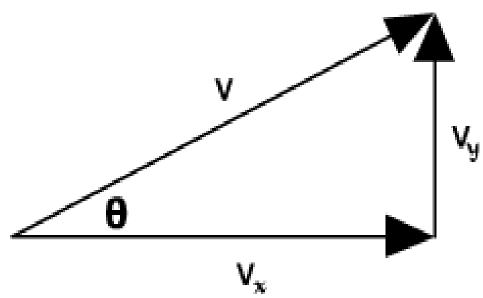
$$Yf = Y_0 + Vay + \frac{1}{2}at^2$$

Acceleration for projectiles near the Earth's surface is -9.8 m/s$^2$ . We don't re-write the equation with a negative sign. Rather we use the negative acceleration value when solving problems.

When a projectile is launched horizontally a ball rolls off a table, a car runs off the edge of a diff, etc. Here the original *y*-velocity is zero for example. If the projectile drops to meters, you can set the $Y_0 = 0$ and $VF = 10$m. Or, you can set $Y_0 = 10$m and $VF = 0$. Either works out the same.

**Velocity :**

To determine the total velocity of projectile, we combine the a horizontal velocity.




('*Vx*') and the vet vertical velocity ('*Vy*') using the Pythagorean Theorem,

$$V = (Vx^2 + Vy^2)^{1/2}$$

**At maximum height :**

At the top of its path the projectile no longer is going up and hasn't started down, yet. Its vertical velocity is zero ($Vy = 0$). The only velocity it has is just its horizontal velocity, *Vx*. Remember, the horizontal speed stays constant throughout the projectile path. A common misconception occurs at the top of a projectile's are: When asked, what the acceleration of the projectile 1s at this point-many people answer "zero". If it were zero, the projectile would simply keep going in a straight line. However, gravity is still acting, pulling it down, and accelerating it, towards the earth. Thus the acceleration at the top is still -9.8m153, just as it's been all along.

**Range of Projectile :**

For a projectile that is launched at a angle and returns to the same right, we can determine the range or distance it goes horizontally fairly simple equation using *a*. However, we will focus on the results of studying that equation rather than solving it here.

- When the projectile is launched at a steep angle, it spends more time in the air, than it does when launched at a shallow angle.
- When the projectile is launched at a shallow angle, it goes faster in the horizontal direction than if it is launched at a steep angle.

The ideal combination of time in the air, and horizontal speed ours at 45°. Thus, the maximum range or distance occurs when the projectile is but had launched at this angle. This applies to long jumpers, and soccer balls that are two good examples. However, if the projectile starts at a point higher than where it lands, the ideal distance doesn't occur at a 45° angle. Ask your instructor for an explanation. If you calculate the range for a projectile launched at 30, you will find it's the same as a projectile launched at 60°. The same goes for 40° and 50°.

The graph of range *VS* angle is Symmetrical around the 45" maximum. The equations used to find out various parameters are shown below:

$$\text{Time of flight } (T) = \frac{2u \sin\theta}{g}$$

$$\text{Maximum height } (H) = \frac{u^2 \sin^2\theta}{2g}$$

$$\text{Horizontal range } (R) = \frac{u^2 \sin 2\theta}{g}$$

If the body projecting from a height, "$h$" above the ground level, the additional height '$h$' is to be considered and the equations modified accordingly.

**Application :**

- A complex from of projectile application in modern life is a rocket or missile.
- Projectile are widely used by sportsman, especially the javelin throw, shot put, discus and hammer throw, etc.
- Projectile are also used in archery and shooting.

**Examples :**

❖ A body projected upwards from the level ground at an angle of 60 with the horizon has initial speed $10\sqrt{3}\ \mathrm{ms}^{-1}$. Find its (a) time of flight (b) horizontal range (c) magnitude and direction of the velocity with which it strikes the ground (d) position after 2sec ($g = 10\mathrm{ms}^2$)

**Solution :**

We have, Inclination with the horizon $\alpha = 60^\circ$

initial speed, $U = 10\sqrt{3}\mathrm{ms}^{-1}$ we know that,

a. $T = \dfrac{2u\sin\alpha}{g}$

$$= \frac{2\times 10\sqrt{3}\ \sin 60^\circ}{10} = 2\sqrt{3} \times \frac{\sqrt{3}}{2} = 3\text{sec}$$

b. $R = \dfrac{u^2 \sin 2\theta}{g}$

$$= \frac{(10\sqrt{3})^2.\sin 2\times 60^\circ}{10} = \frac{300 \times \sin 120^\circ}{10} = 30 \times \frac{\sqrt{3}}{2} = 15\sqrt{3}\ \text{m}$$

c. Let $v$ be the striking velocity making angle $\theta$ with horizon.

Now, $Vx = V\cos 60° = 10\sqrt{3} \times \frac{1}{2} = 5\sqrt{3}$ m/sec.

$Vy = V\sin 60° = 10\sqrt{3} \times \frac{\sqrt{3}}{2} = 15$ m/sec.

Then,

$r = \sqrt{Vx^2 + Vy^2}$

$= \sqrt{(5\sqrt{3})^2 + (15)} = \sqrt{300} = 10\sqrt{3}$ m

and

$\tan\theta = \frac{Vy}{Vx} = \frac{15}{5\sqrt{3}} = \sqrt{3} = \tan 60° \ \therefore\ \theta = 60°$

d. Let $(x, y)$ be position of the body.

Then,

$X = u\cos 60° \times t$

$= 10\sqrt{3} \times \frac{1}{2} \times 2 = 10\sqrt{3}$m

$Y = u\sin 60° \times t - \frac{1}{2} gt$

$= 10\sqrt{3} \times \frac{\sqrt{3}}{2} \times 2 = \frac{1}{2} \times 10 \times 2^2 = 20$m

$\therefore$ Its position = $(10\sqrt{3}$m, 20m)

❖ Find the velocity and the direction of projection of a shot which passes in a horizontal direction just over the top of a wall which is 250m off and 125m high. ($g = 9.8$ms$^{-1}$)

**Solution :**

Let $u$ and $a$ be velocity and angle projection respectively.

Since the shot passes in a horizontal direction just over the top of the wall, so height of the wall is the greatest height attained by the shot.

$$\therefore \quad H = 125\text{m}$$

Also, horizontal range, $R = 2 \times 250 = 500\text{m}$

Now, $$H = \frac{u^2 \sin^2\alpha}{2g}$$

$$\Rightarrow \quad 125 = \frac{u^2 \sin^2\alpha}{2g} \tag{1}$$

Again, $$R = \frac{u^2 \sin 2\alpha}{g}$$

$$\Rightarrow \quad 500 = \frac{u^2 \times 2\sin\alpha \cos\alpha}{g} \tag{2}$$

Dividing (1) by (2),

$$\frac{125}{500} = \frac{\dfrac{u^2 \sin^2\alpha}{2g}}{\dfrac{u^2 \times 2\sin\alpha \cos\alpha}{g}}$$

$$\frac{1}{4} = \frac{\sin\alpha}{4\cos\alpha}$$

$$\tan\alpha = 1$$

$$\therefore \quad \alpha = 45^\circ$$

**Conclusion :**

The interesting applications of parabola involve their use as reflectors and receivers of light or ratio waves. For instance, cross sections of car headlights, flashlights are parabolas where in the gadgets are formed by paraboloid of revolution about its axis. Many real-world situations can be represented by ellipse, including orbits of planets, satellites, moons and cornets, shapes of boat keels, rudders, and some airplane wings.

Some versions of the latest PC monitors and also some televisions came with curved monitors. This adaptation makes the user's eyes effortlessly discern details on the screen compared to flat monitors. Comparing these monitors with flat pics, these curves are hyperbolic. Even in the design of these displays, the manufactures employ hyperbolic estimations.

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