Applied Science Periodical **ISSN 0972-5504**
Volume - XXIV, No. 4, November 2022
Journal website: www.internationaljournalsiwan.com
ORCID Link: https://orcid.org/0009-0008-5249-8441
Google Scholar: https://scholar.google.com/citations?user=BRweiDcAAAAJ&hl=en
Refereed and Peer-Reviewed Quarterly Periodical

Some New Applications of Calculus to Non-linear Sciences

by **Suresh Kumar Sahani[1],** *Department of Mathematics, MIT Campus, Tribhuvan University, Janakpurdham - 45600, Nepal*
E-mail : sureshkumarsahani35@gmail.com
Kripa Sindhu Prasad[2], *Department of Mathematics, Thakur Ram Multiple Campus, Tribhuvan University, Birgunj, Nepal*
E-mail : kripasindhuchaudhary@gmail.com

(Received: October 26, 2022; Accepted: November 15, 2022; Published Online: November 30, 2022)

Abstract:

In this research note we have developed a set of applications for calculus, which are more biology oriented. These include growth/decay problems in any organism population, gene regulation and dynamical changes in biological events such as monitoring the change of patient's 'temperature' along with the medications.

Keywords : Derivatives, biology, growth/decay, temperature, etc.

Objectives / Limitations :

The main motive to prepare this work is to visualise the capacity of the individual and increase the affinity to achieve the certain goals.

a. Derivatives contracts are exposed to high degree of risk due to high volatile price of underlying securities.

b. There is a possibility of default on the part of counter-party in case of derivatives traded over the counter due to lack of due diligence process. OTC derivatives as compared to exchange derivatives lacks a benchmark for due diligence. This is one of the major drawbacks in trading of derivative instruments.

c. Investor's requires high knowledge and expertise for trading in these instruments as compared to other securities likes stocks and metals.

d. Derivatives are instrument which are used for speculation purpose for earning profits. Sometimes huge losses may occur due to unreasonable speculation as derivatives are of unpredictable and high risky nature.

Introduction :

Derivative, itself a ocean and cannot be elaborated by single mathematician. Hence it is a part of calculus (Latin name of stone), in which calculus means calculation in Roman civilization. Calculus, known in its early history as infinitesimal calculus, is a mathematical discipline focused on limits, functions, derivatives, integrals, and infinite series. Isaac Newton and Gottfried Leibniz independently discovered calculus in the mid-17th century. However, each inventor claimed the other stole his work in a bitter dispute that continued until the end of their lives.

Definition :

There are so many definition of Derivative. Some of them are refers as shown below:

a. Derivative refers to such a mathematical calculating device from which we can able to calculate the rate of small change in any population, locality, biome, etc.

b. Derivative can be defined as to estimate such a value for any sort of potential by small change (decrement or increment).

c. We have developed a set of application examples for Calculus, which are more biology oriented. These include: growth/decay problems in any organism population, gene regulation and dynamical changes in biological events such as monitoring the change of patients' temperature along with the medications.

Discussion :

(Problem No. 1) :

In a village having unlimited source of population "N_0", the population becomes "N_t". What is the change in population size after time "t" ?

Here,

Original population $= N_0$

after time $= t$

The final population becomes $= N_t$

We know that for exponential growth model as there is unlimited supply of source.

We have to find $\left(\frac{dN_0}{dt}\right) = ?$

Let B and D be the total number of birth and death and no migration takes place then,

$$\text{Birth rate } (b) = \frac{B}{N_0} \qquad (1)$$

$$\text{Similarly, death rate} = \frac{D}{N_0} \qquad (2)$$

$$\text{As we know, } \left(\frac{dN_0}{dt}\right) = B - D \qquad (3)$$

From 1, 2 and 3, we can write;

$$\left(\frac{dN_0}{dt}\right) = bN_0 - dN_0$$

Or, $$\left(\frac{dN_0}{dt}\right) = (b - d)N_0$$

Here $(b - d)$ is intrinsic rate of natural increase so we can write it as "r"

Hence

$$\left(\frac{dN_0}{dt}\right) = rN_0 \text{ Ans.}$$

On integrating the above expression, we can write

$N_t = N_0 e^{rt}$ which is required population size.

(Problem No. 2) :

Suppose the flow rate of blood in a coronary artery has been reduced to half its normal value by plaque deposits. By what factor has the radius of the artery been reduced, assuming no turbulence occurs?

Find the velocity gradient at $r = 1\text{mm}$ $\begin{cases} \textit{radius of coronary art.} = 2.2mm \\ \textit{velocity of blood} = 0.0027Ns/m^2 \\ \textit{length of vessel} = 20mm \\ \textit{and pressure} = 0.05N \end{cases}$ for this artery. Does he have to refer for further examination or treatment by Angiologist?

We know that, from laminar flow and Poiseuille's law,
we can write that

$$Q = \frac{(P_1 - P_2)\pi r^4}{8\eta l} \qquad (1)$$

We need to compare thew artery radius before and after the flow rate reduction.

With a constant pressure difference assumed and the same length and viscosity, along the artery we have;

$$\frac{Q_1}{r_1^4} = \frac{Q_2}{r_2^4}$$

So that $\quad Q_2 = 0.5Q_1$

Similarly, $\quad r_2^4 = 0.5r_1^4$

Therefore, $\quad r_2 = (0.5)^{0.25} r_1$. Hence the artery decreases by 16%.

Again,

On differentiation equation 1, we can write,

$$Q^1 = \frac{-2rp}{4\eta\iota} \qquad \text{Or, } Q^1 = \frac{-2\times 10^{-3}\times 0.05}{4\times 0.0027\times 2\times 10^{-3}}$$

Or, $Q^1 = -0.46\text{m/s}$

Hence, if the gradient is too high then the person may has a constriction and has to be referred for further examination but it is negative so he has no any problem and no need of angioplasty.

(Problem No. 3) :

Find the rate change of a tumor, when initial volume is 10 cm^3 with growth constant 0.075 at the period of 7 years.

Let us assume and from given;

V_0 = Initial volume = 10cm^3

e = exponential growth = 2.718281282828.....

k = growth rate = 0.075

t = time = 7 years.

In order to find the change in tumor growth the derivative of volume equation becomes [$v(t)$] refers to volume as function of time.

We can write,

$$V(t) = V_0 . e^{Kt}$$

Then $V^{|}(t) = \frac{d}{dt}(V_0 . e^{Kt})$

Because e^{Kt} is a complicated function, we can use derivative chain rule to derivate it.

$$e^{Kt} \text{ put } u = e^{Kt} \text{ then, } \frac{du}{dt} = e^u \tag{1}$$

then, $y = e^u$

or, $$\frac{dy}{dt} = k \qquad (2)$$

From Chain rule,

we know, $$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt}$$

or, $$\frac{dy}{dt} = k.\, e^{u} = k.e^{kt}$$

Now, putting the value of the known values we get;

$$V(7) = 10.2 \times (7)^{0.075 \times 7}$$

Therefore, $v(t) = 15.03\text{cm}^3$ Ans.

(Problem No. 4) :

The height of a child as a function of time can be modelled with the equation $h(t)\,\frac{3t^2}{t^2+4}$, h is height in unit centimetre (cm) and in time t sec(s).

i. How tall is the child in five month?

From above equation we can find $h(5) = ?$

We know;

$$h(5)\,\frac{3.5^2}{5^2+4} = 2.59 \text{ cm}$$

ii. At the same month mark, what is the child's growth rate?

$$\text{Growth rate} = \frac{\Delta h}{\Delta t} = h^1(t)$$

$$\text{Or, } h^1(t) = \frac{(t^2+4)(6t) - 2t(3t^2)}{(t^2+4)^2}$$

$$\text{Or, } h^1(t) = \frac{6t^3 + 24t - 6t^3}{(t^2+4)^2} = \frac{24t}{(t^2+4)^2} \qquad (1)$$

Now $h^1(5) = \frac{24.5}{(5^2+4)^2} = 0.143$ Ans.

iii. What is the change in growth rate of that child?

Change in growth rate is derivative of equation 1

Or, $\frac{dh^1}{dt} = \frac{d}{dt}\left(\frac{24t}{(t^2+4)^2}\right)$

$$\text{Therefore } \frac{dh^1}{dt} = \frac{24[t^2+4-4t^2]}{(t^2+4)} \qquad (2)$$

Now $h^{11} = \frac{24[5^2+4-45^2]}{(5^2+4)} = 0.0699$ m/five month.

(Problem No. 5) :

A Patient who is suffering from viral influenza, came to OPD and along him (v_0) other cases are also seen in health sciences. On examination, among them "C" of them were dead.

(i) How much concentration of people are seen after t days?

Here, total no of infected person = v_0

And, death number = C

Then we have to find the small change in viral concentration $\left(\frac{dv}{dt} = ?\right)$

Then, $v_0 \times -c = \frac{dv}{dt}$

Or, $-cv_0 = \frac{dv}{dt}$

On integrating the equation we can write,

$$-ct + A = \text{In}[v(t)]$$

$v(t) = e^A . e^{-ct}$, in initial concentration, $(t) = 0$ so we can write that;

$e^A = v(0) = v_0 =$ initial concentration so we have,

$$v(t) = v_0 . e^{-ct} \quad \text{Ans.} \tag{1}$$

(ii) During chronic stage of influenza, Uninhibited rate of viral production cell per day?

As per the above equation 1, we get;

Let P be the uninhibited rate of viral production in viral load as constant.

Then, we can write the equation (1) as;

$$\frac{dv}{dt} = P - Cv_0 \tag{2}$$

As, it is in chronic stage, $\left(\frac{dv}{dt} = 0\right)$

Then equation 2 becomes, $p = Cv_0$ Ans.

(Problem No. 6) :

For a neurone modal, if the concentration of ion are in order of $(ion)_{out} >> (ion)_{in}$. In the membrane of neurone, Estimate the external current applied on it (I^{app}).

Here, From Nernst potential equation as. $E_{ion} = \frac{RT}{zF} ln \frac{(ion)_{out}}{(ion)_{in}}$. For a rough information that the membrane contain so many ions like Na^+, Cl^-, A^-, etc. so,

Ionic current $(.I_{ion} = . g_{ion}(V - E_{ion})$

For external current $(I^{app}) = ?$

We have,

Derivating the expression for capacitor,

$$\frac{dI_C}{dt} = C\frac{dV}{dt} \tag{1}$$

Similarly,

We can use Kirchhoff's current law,

Or, $$C\frac{dV}{dt} + \sum_{ion} I_{ion} + I_L = I^{app}$$

Then, we can write for human neuron cell,

$$C\frac{dV}{dt} + g_{Na}(V - E_{Na}) + g_K(V - E_K) + I_L = I^{app} \quad \text{Ans.}$$

Conclusion :

As per the above discussion; according to us, derivative plays a great role for almost every sector of the cosmos. Even in science (field of medicine, para-medical like pharmacy, forensics, and so on), computer (binary system, C-programme), etc. are also spectra of derivative. From above discussion, we had learnt that if to evaluate the size of tumor or to check the patient having some sort of problem in blood vessels and population related concern are being estimated.

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