The Mathematics Education **ISSN 0047-6269**
Volume - LVII, No. 2, June 2023
Journal website: www.internationaljournalsiwan.com
ORCID Link: https://orcid.org/0009-0006-7467-6080
Google Scholar: https://scholar.google.com/citations?hl=en&user=UOfM8B4AAAAJ
Refereed and Peer-Reviewed Quarterly Journal

Some New Studies on Logic and Sets

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(Received: April 10, 2023; Accepted: May 25, 2023;
Published Online: June 30, 2023)

Abstract :

In this report, we have studied the concept of Logic and Set. Logic and Set, which has a brief and huge history is compressed in a short form and their various definitions as well as theorems have been mentioned by us in the report. A large number of unrealisable uses of Logic and Set can be seen in this world, among which various examples of the uses of Logic and Set are presented by us. Many Mathematicians and Scientists who have worked hard and contributed a lot in the fields of Logic and Set has inspired us. The Mathematician like George Boole, Georg Cantor, and Gottlob Frege has inspired us to present the brilliant examples of uses of Logic and Set in Mathematics. Various Scientists like Albert Einstein, Stephen Hawking and Galileo Galilee and their achievements

motivated us to study their works and find out how they used Logic and Set in their works. The father of computer Charles Babbage, first programmer Lady Augusta Ada Lovelace as well as the advancement of computer made us to rethink the uses of Logic and Set in the sector of Computer as well as programming. While studying these uses of Logic and Set in different fields, a thought suddenly came in our mind that how our ancestors used and we are still using the concept of Logic and Set in our daily life. Thus all these Mathematicians, Scientists and concept of Logic and Set inspired us and helped us to complete the report.

Objectives :

The main objective is to study the use of 'Logic and Sets' and to find out their application in different fields of sciences.

The application of Logic in different fields are:

- Mathematics
- Computer Science
- Physics
- Economics
- Biology
- Psychology
- Astronomy
- Population Study and Growth
- Daily Life

Similarly the application of Sets in different fields are:

- Mathematics
- Computer Science
- Astronomy
- Population Study and Growth
- Daily Life
- Manufacture and Storation of goods
- Computer Engineering

Limitations :

The main objective is to study only a specific part of uses of Logics and Sets in different fields which is

- To study the basic uses of Logics and Sets in Mathematics,
- To study how Logics and Sets have been a very basic aspect in studying Astronomy,
- To study how Logics and Sets is used in Computer Science,
- And finally to study the basic uses of Logics and Sets in our Daily Life.

Introduction :

Historical Background of Logics and Sets :

Logics are very common thing used in the world. It is also called Elementary Logic. The word Logic is derived from the word "logos" which means reason. Logic was developed independently and has been used by humans from very old time. Different famous people like Aristotle, Avicenna, William of Ockham, etc. have contributed in the development of Logics. It is the base in evolution and development of different study areas and different fields.

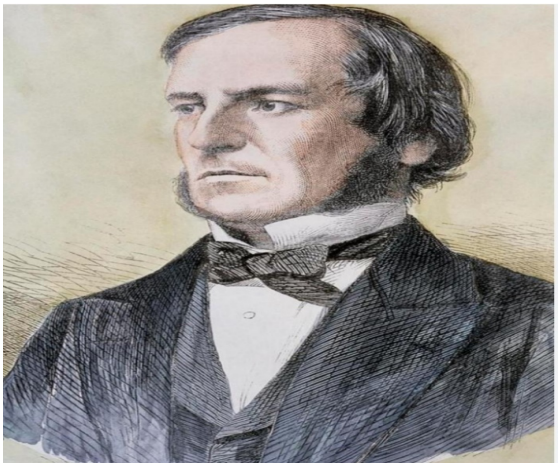
The word "Set" is known to carry out the meaning of words like collection or class. Sets has been a basic thing in human context. Since the 5^{th} century BC, different people have contributed in development of Sets. The concept of Set is based on the 'Set Theory' which was given by a German Mathematician Georg Cantor (1845-1918). The concept of Set and Set Theory is used in advancement of different fields (see [1-5]).

Logics and Sets have a brief history in different fields. Some of the fields are :

1. Mathematics :

George Boole (1815-1864), who was a professor of mathematics at Queen's College, Cork, first used Logic in mathematics. Mathematics, which has always been an integral part of human life since the origin of humans, became much easier to understand with the introduction of Logic. Later on, many people like Gottlob

Frege, Alfred North Whitehead and Bertrand Russell made significant contribution in Mathematical Logic. With the advancement of the concept of Logic, it has been used in every field. Logic has been used to prove all mathematical theorems especially the theorems like set theory, model theory, etc. and to solve all the numericals.



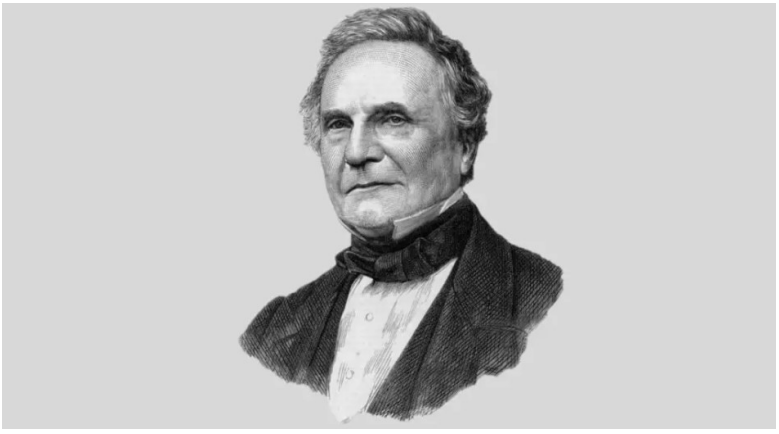
Similarly, Set has also been a basic part of Mathematics. Set Theory or Set is said to be the foundational system for the whole mathematics. Introduction of Set in Mathematics has made the study of Mathematics very easy. Set is used at every point while studying mathematics. Mathematics, which has always been an integral part of human life since the origin of humans, became much easier to understand with the introduction of Sets. Numbers, the most basic concept in Mathematics is arranged in different categories such as Rational and Irrational Numbers, Integers, Whole Numbers, Real Numbers, etc. These categories can be represented in form of Sets.

2. Astronomy :

Like mathematics, Astronomy has also been an integral part of human life since the origin of humans. It is said that around 1000 BCE, humans started study Astronomical Objects and with the start of study of Astronomical Objects we apply logics while studying Astronomical Objects. The introduction of Logic has helped to study many parts of Astronomy and find out many conclusions. The concept of Constellations was discovered by applying logic. The age of our universe, existence of black holes and searching different cerestial bodies in the universe has been possible by applying simple logics in Astronomy.

Similarly, Sets also has a basic importance in Astronomy. With the start of study of Astronomical Objects, people needed to remember the objects and name them. Thus, the objects were arranged in form of different sets. This process is still in practice in a broader form where scientists arrange the objects in different sets as category of stars, planets, etc. Set was also used in the concept of Constellation.

3. Computer Science :



Unlike Mathematics and Astronomy, humans have started study of computer science very recently. Development of Difference Engine and Analytical Engine by Charles Babbage started the development of modern computers. Computer Science has a very important part to study called as Boolean Logic or Boolean Algebra. Logic was introduced by George Boole. Logic is used in Boolean Algebra to design Logic circuit, Binary devices and to find the errors available in Binary devices. Logic gates were the reason for the development of microchips. The inverters, burglar alarms, double switch equipments were developed on the basis of Logic gates. Evolution of Programming was also based on Logic.

Similarly, Set is also used in Boolean Algebra. Set is used in form of venn diagram to represent the result of the Logic Circuits which help us to get know the possibilities of results of the Logic Circuits. Set has been used to represent the result of appliances like burglar alarms, automatic heater, double switch equipments, etc. Set and Set Theory has been largely used in Database Programming, C-programming, Q-basic and in their development.

4. Daily Life :

Logic is a integral part of our daily life. Since the origin of Human civilization, Logics has been used by humans to carry out our works. Logic was developed independently in several cultures. We use simple logics or even think logically what we should do. We used Logic to deal with any problem and situation. We apply logics to find the reason of different events happened, to predict what will happen near or a long time in the future. Thus Logics has been a part of human life.

Similarly, Set is also a basic thing in our Daily Life. Humans from their origin, lived in group or set of around 20 to 30 people and they also categorized people on the basis of their works as subsets. This is still prevailent in a changed format. Also we can see that since the origin of Human civilization, humans use sets to carry out the basic works. Humans used Sets to arrange objects we use in different groups on the basis of their uses. The primitive humans arranged the agricultural tools together and the weapons together. This is still prevailent in humans in a broader form.

Definitions of Logics and Sets :

Various Definitions of Logics and Sets has been given.

For Logic :

- Logic is traditionally defined as the study of the laws of thought or correct reasoning.
- Logic is also defined as study of valid arguments and the science of logical truths.
- Logic is a formal science that studies how conclusions follow from premises in a topic-neutral way.
- Logic is a discipline which deals with the study of language of valid reasoning.

For Set :

- Set can be defined as a well-defined list or collection of material objects such as books and pens or conceptual objects such as numbers and points.

- Georg Cantor, one of the founders of Set Theory defines set as "A Set is a gathering together in a whole of definite, distinct objects of our perception or our thought which are called elements of the set.
- Bertrand Russell called a set a class: "When mathematicians deal with what they call a manifold, aggregate, Menge, ensemble, or some equivalent name, it is common, especially where the number of terms involved is finite, to regard the object in question (which is in fact a class) as defined by the enumeration of its terms, and as consisting possibly of a single term, which in that case in the class.
- Roaster or Enumeration notation defines a set by listing its element between curly brackets seperated by commas.
- Naive set theory defines a set as any well-defined collection of distinct elements.

Statements :

There are many theories based on sets and logics.

For Logics there is not any theory but there are some laws :

- Idempotent Laws:
  a. $p \vee p \equiv p$
  b. $p \wedge p \equiv p$
- Commutative Laws:
  a. $p \vee q \equiv q \vee p$
  b. $p \wedge q \equiv q \wedge p$
- Associative Laws:
  a. $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$
  b. $p \vee (q \vee r) \equiv (p \vee q) \vee r$
- Distributive Laws:
  a. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  b. $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- De-Morgan's Laws:
  a. $\sim (p \wedge q) \equiv (\sim p \vee \sim q)$
  b. $\sim (p \vee q) \equiv (\sim p \wedge \sim q)$

For Sets there is :

- **Set Theory :** Set Theory is the branch of mathematical logic that studies sets, which can be informally described as collection of objects.

  Beside Set Theory there are some laws when 3 sets A, B and C are subset of a Universal set U:

- Idempotent Laws: Union or Intersection of a set with itself is equal to the set.
- Commutative Laws: The Union of set A and set B is equal to the Union of set B and set A. Also the Intersection of set A and set B is equal to the Intersection of set B and set A.
- Domination Laws: The Union of set A with universal set U is equal to the universal set U. Also the Intersection of set A and an empty set is equal to empty set.
- Absorption Laws: The Union of set A and another set which is Intersection set of set A and B is equal to set A. Also the Intersection of set A and another set which is Union set of set A and B is equal to set A.
- Associative Laws: The Union of set A and Union set of set B and C is equal to the Union of Union set of A and B and set C. Also the Intersection of set A and Intersection set of set B and C is equal to the Intersection of Intersection set of A and B and set C.
- Distributive Laws: The Union of set A and Intersection set of set B and C is equal to the Intersection of Union set of A and B and Union set of A and C. Also, the Intersection of set A and Union set of set B and C is equal to the Union of Intersection set of A and B and Intersection set of A and C.
- Complementation Laws: The Union of set A and Complement set of set A is equal to Universal set U. Also Intersection of set A and Complement set of set A is equal to Empty set. This law also explains that Complement set of Empty set is equal to Universal set U, Complement of Universal set U is equal to Empty set and Complement set of Complement of set A is equal to set A.

- De-Morgan's Laws: The complement of Union set of set A and B is equal to Intersection set of Complement sets A and B. Also, The complement of Intersection set of set A and B is equal to Union set of complement sets A and B.

Discussions :

Logics and Sets are used in various fields as there are many examples. Some examples of uses of Logics in different fields are given below:

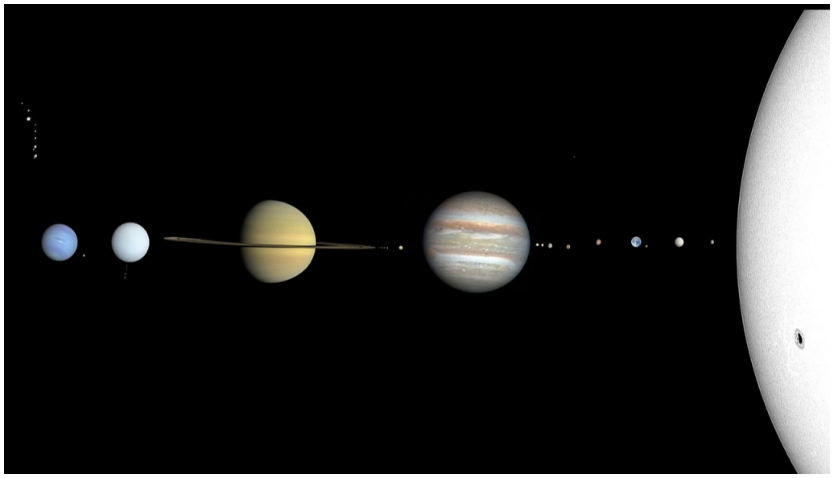
- **Mathematics :**

Logic is used in mathematics to get simple results. We logically can say if 2 triangles are congurent then their areas are equal. We can also say if a real number is divisible by at least 3 real numbers then the number is composite number. Beside these, logic is used in different areas of maths like proving Theorems. Logic is applied in mathematics to prove many theorems like Set Theory, Model Theory, etc. We apply Logic to solve any mathematical problem or to prove any theory in mathematics. Even sets can't be made without applying any logic.

Set is used in Mathematics as a basic concept. We study numbers in form of sets (such as Set of real numbers = {1, 2, 3, 4, 5, …..} and Set of integer numbers = {…., -3, -2, -1, 0, 1, 2, 3, ….}) which makes easy to study the base of mathematics. Without using sets to classify numbers, it would have been very difficult to study mathematics and solve mathematical theories. Beside these Set is used in many other field and topics in mathematics like to find probability of happening of an event where the number of overlapping circles are probability of the same number of events and the overlapping part represents the common probability in the events.

- **Astronomy :**

Logic is also used in Astronomy as a basic concept. We apply simple logics while studying Astronomy. It has helped to find the real age of universe (we estimated the real age of universe, but it was proved when we found that all stars and galaxies are less than the estimated age of universe). Thus by logic, we found the real age of Universe. Also we discovered Black holes without their visual observation just by applying the logic that they affect the objects around them. So we studied the effects on cerestial objects and find the presence of black holes near the objects. Also it is used to study Stars, Galaxies and cerestial objects. We also used Logic in the concept of Constellation. Constellations were used by humans in history for finding directions in the night sky and agricultural purposes. Humans applied logic in the night sky and imagined a pattern or outline representing an animal, object or any mythological subject etc and concluded the pattern as constellation. Thus Concept of Logic gave the Concept of Constellation. For example: Orion constellation is a pattern as a Greek mythological hunter Orion and Cancer constellation is a pattern like a Crab.



Set is basically used while studying Astronomy. Sets are used to divide Planets, Stars, Galaxies, Nebulae in different categories on the basis of their appearance, location and other characteristics as (Set of galaxies in our local group will have Milkyway, Andromeda, Triangulum and a finite number of other galaxies as set elements. Similarly set of planets in out solar system will have Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune). Without doing so, the study of planets,

stars and space would have been a tedicious task where we would have to remember all individual stars, planets and galaxies, etc. and individual characteristics of each of them. Set was also used in concept of constellation. Humans grouped a number of stars according to their pattern as a set. For example: The pattern of Orion Constellation is based on a set of 7 stars.

- **Computer Science :**

Logic's Truth Table is used to represent the result of Logic Circuit and Binary Devices. Different Logic Circuits such as AND, OR, NOT, NAND, XOR, etc. Gates are created on the basis of laws of Logic and are represented by Truth Tables. The truth tables of Logic and Logical gates can be seen identical to each other. The result is based on the 2 input signals and the arrangement of the circuit. More number of Gate operations enables computer to do more complex operations. Logic is further used with Logic Circuits to manufacture microchips and the equipments like automatic sensing heater, doorbells, inverters, motorcycles, etc. By installing an inverter system, electrical appliances can run on any one between the main electric supply and the inverter which is based on the concept of OR Gate. The motorcycles also can be started by any one between kick start and self start which is also based on OR Gate. Automatic Sensing Heaters run on concept of NOT Gate where if temperature rises more than a set temperature then it stops and works when temperature is lower than set temperature. Beside Logic Circuit, Logic has been used in Q-basic as well as C-programming. Logic is used to determine the results in Q-basic and C-programs. We use Selection Control Statements and Looping statements which includes one or more conditions and result is printed by applying logics in conditions as 'true' and 'false'. This type of use of logic is also applied in Database Programming at any level.

Sets has also been used as a representation of results of Logic Circuit. The Gates are represented in venn-diagrams too. The venn-diagrams show which results are given by a particular Gate. It has 2 overlapping circles representing the 2 input signals and overlapping part is the common result of 2 input signals. Set is also used to represent the result of Logic Circuits in different appliances. By installing an inverter system, electrical appliances can run on any one between the main electric supply and the inverter which is based on the concept of OR Gate. Results are as venn-diagram where the condition when supply is ON is the universal set only

and condition when supply is ON is represented in the sets. The motorcycles also can be started by any one between kick start and self start which is also based on OR Gate whose results are universal set only for no try and the main sets represent the condition of applying start. Automatic Sensing Heaters run on concept of NOT Gate where if temperature rises more than a set temperature then it stops and works when temperature is lower than set temperature. This can similarly be represented in form of Set by applying NOT Gate rules. Set is largely used in Q-basic and C-Programming. Set forms the basis of many data structures used in programming. Set theory determines which data will be included and excluded in search and selection while programming. Also there are folders where we can store different types of files, which is based on concept of Set. We can store some specific files in a set called folder where the files may or may not related to each other.

Daily Life :

We as humans, have the habit to apply Logics in our daily life. From doing a work to finding something, Logics can be applied anywhere. We predict future by applying logics (if the weather is hot and humidity is high then logically we conclude that it can rain in the future). We predict about other people (if they bought a car they must be rich). None of the works is performed by us without applying Logics. Beside these we apply logics in various part of our daily life. Applying Logics in our daily life helps us to live our life easier.

Sets is also used by Humans in their daily life. We live between a set of people either in our home as family, students as school, employees in job office, etc. We mostly keep objects, tools, equipments, etc in Set (Kitchen is an example of set where we mostly find only the objects to cook and eat like Pan, Plates, Spoons, Utensils, etc in a set. Wardrobe is also an example, where we mostly keep clothes in a set. We study different subjects like Physics, Chemistry and Mathematics which is also a set of common type of topics to be studied together). Beside these we use the concept of sets in various part of our daily life. Thus, living life by keeping everything in sets is a better way to live.

Thus by studying Logic and Set, I get to know that I have been using the concept of Logic and Set without getting known about this. I apply simple Logic as whenever I see that there are dark clouds in sky, I predict that it could rain and I take

an umbrella with me. Whenever it rains, I ride cycle very carefully as I predict roads are slippery. Whenever I store fruits, vegetables and cooked foods, I keep them in refrigerator as I predict they could get rot in open environment. On Diwali, I don't keep burning deepak or candles near to clothes or flammable items as they could catch fire. Whenever I buy any product, I see the durability, efficiency with respect to cost of the product as it can be a good deal. I see both the advantages and disadvantages of any object of work and then choose what to do by comparing their advantages and disadvantages. If they have more disadvantages then I conclude that this is not a good deal but if they have more advantages then this is a good deal. When I open whatsapp or messenger, I can see the upper messages are the newest and messages gets old as I scroll down. I eat more healthy food as it contains more nutrients and eat less the junk food as they have no any nutrients. All these real life things and events are the examples of Logics. In these things and many other things, I apply Logic which helps me to live a better life.

I apply the concept of Set as whenever I need to place things like Utensils, Fruits and Clothes somewhere, I see to which place they can belong. Like Utensils are placed in Kitchen, Fruits are placed with other fruits mostly in Refrigerator and Clothes in wardrobe. This is based on concept of sets where Kitchen is the set of materials needed for cooking, Refrigerator is a place where we can store Fruits and Vegetables in sets and Wardrobe is set of clothes and things we wear. Whenever I go to buy anything, I notice that there are separate shops for separate type of products. So I go to vegetable shop to buy vegetables and clothes shop to buy clothes. I never realised that actually I am a partof a set of people who I consider my family. I am a singleton subset of a set called family, students, locality, etc. and of a larger set called as citizen and of a Universal set called Humans. When I use youtube, I keep different playlist of videos for different videos like songs in songs playlist, comedy videos in different playlist and informative videos in different playlist. I study my syllabus without noticing that different topics are placed in different sets on basis of their similarities and differences as topics to study chemicals and elements are together as chemistry, topics to study different physical forces, heat, light, etc. are together as physics and topics to study english grammar and english speaking skill are together as english. All these real life things and events are based on concept of Sets. In these things and many other things, I apply concept of Sets which helps me to live a better life.

Conclusion :

After discussing Logic and Set, we get to know many uses of Logics and Sets in different fields and in our Daily Life.

- We get to know how Logic and Set are a basic concept for the whole Mathematics. We found that Mathematics is incomplete without Logic and Set.
- We get to know how Logic and Set is important while studying Astronomy as well as Astronomical objects and got to know how difficult it would be to study Astronomy without using Logic and Set.
- We find how Logic and Set are connected with the Computer Science, concept of Boolean Algebra and with the concept of computer programming.
- We discovered the real life uses of Logic and Sets which we always use in our daily life without noticing this and without getting acknowledged about this.
- We get to know the uses of Logic and Sets in many different fields like Physics, Biology, Population Growth and Study, Manufacture and Sortation of goods, etc.
- I get to know how I was using concept of Logic and Set in my everyday life without getting known.

Logics and Sets are being used in almost every field in a large part. These topics has contributed to make our life easier. It is good to use these topics in every field but like excess of everything is harmful, similarly overusing Logic and Set in every field will again cause negative advantage. Using Logics more than required always keep us in illusion or Maze of Logics and drives us away from reality. Also using Sets more than required is harmful. The most basic use of Set is to divide work or objects in groups and reduce the effort required. Doing the process upto an extent will make it easier but after an extent it will again make the work require more effort (for example: if we categorize stars into different fields on basis of every difference between them then it will make the study of stars even more difficult).

So Logic and Sets should be used but not be overused. Logic should be used in every field but it should not be more than required. Similarly Set should be made for some vast difference but not for every minor differences. If this work will be applied to every field then the work in every field required would be the least and it will make both the field and our lives easier.

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