

A Note on Some Problems in Mathematics

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Abstract :

In this note, we mainly provide two mathematical problems and discuss their kinship with the well known Koethe problem. The first problem is based on the theory of non-commutative matrices and the second problem is based on the theory of non-commutative rings. These problems appear to be of fundamental importance in mathematics.

Keywords : matrix, nilpotent, non-commutative ring, open problem, Koethe problem.

MSC 2010 : 12xx, 97H40, 13A99.

1. Introduction :

There are several problems and conjectures in mathematics which remain open for several years. For example one may refer [1-2] for a well known open problem introduced in 1930 which is widely known as the Koethe problem.

In this note, we provide two problems. The first problem is based on the theory of non-commutative matrices [3]. We generalize the first problem to get the second problem and the second problem is based on the theory of non-commutative rings [4].

These problems appear to be of fundamental mathematical importance and therefore these deserve to get the attention.

2. Two Mathematical Problems :

Problem 1 :

Let A and B are two non-commutative matrices such that AB is a non-zero nilpotent matrix. Similarly let C and D are two non-commutative matrices such that CD is a non-zero nilpotent matrix and $AB + CD$ is a nilpotent matrix. Is $BA + DC$ nilpotent?

Remark : Non-commutative matrices play very important role for the theory of non-commutative rings. The notion of nilpotent elements in ring theory has kinship with the notion of nilpotent matrices in the theory of matrices. Therefore problem 1 can be naturally generalized as a problem of the ring theory as follows.

Problem 2 :

Let R is a non-commutative ring. Let pq and rs are any two non-zero nilpotent elements of R such that $pq + rs$ is nilpotent. Is $qp + sr$ nilpotent?

3. Significance and Discussion :

The second problem is just a generalization of the first problem from the theory of matrices to the theory of rings. A negative answer to the first problem may suggest a negative answer to the second problem. These are of fundamental mathematical importance as these appear to have some connection with the well known Koethe problem which is given as follows. Is every nil left (right) ideal contained in a two sided nil ideal [1]? Though Koethe problem seems simple but in spite of several attempts by several mathematician world wide it is still an open problem in mathematics in general and in ring theory in particular. Similarly the above two problems presented in this note seem simple but both appear to have deeper mathematical insight as they seem to have some sort of kinship with the long standing Koethe problem. It may be speculated that a negative answer to the problem 2 may lead to a negative answer to the Koethe problem.

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