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Nocturnal decay of peak ionospheric density at a midlatitude location during sunspot minimum equinox conditions: Closed form solutions

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Abstract:

The ionosphere is that part of the upper atmosphere where ions and electrons are present in sufficient quantities to affect the propagation of radio waves. It is a vast region having multi-faceted properties and variations which comprises a subject matter by itself. All scientific studies of the ionosphere invariably require global, regional or local numerical modeling. In this paper, we study a limited aspect of the ionosphere using analytical exact solutions in closed forms. Specifically, we analyze the nocturnal decay of the ionospheric peak electron or ion density above Arecibo, Puerto Rico during sunspot minimum and equinox conditions. Two kinds of loss processes are recognized:

(1) a quadratic ion-electron recombination reaction; and (2) a linear ion-neutral charge transfer process. The nocturnal decay of the peak electron density is studied under each loss process separately, and compared with that obtained from a numerical model. It is found that the ion-neutral charge-transfer is the dominant loss process, nine times more effective that the ion-electron recombination process.

Introduction:

The ***ionosphere*** has been defined as that part of the upper atmosphere where ions and electrons are present in sufficient quantities to affect the propagation of radio waves. The ***solar extreme ultraviolet radiation*** is the main source of ionization in the ionosphere with x-rays, cosmic rays and meteor showers comprising minor sources. The ionosphere consists of several distinct regions. The dominant part of the ionosphere is called the ***F region*** which lies roughly in the 150 to 600 km altitude. It is here that the electron density reaches its peak. The peak electron number density labeled ***NmF2***, generally lies around 250 km altitude during the day but moves higher to a more constant value of 300 km altitude. This altitude is labeled as ***HmF2***. The dominant ion in the F region is O^+. Owing to the ***charge-neutrality condition***, the ***electron density*** is equal to the ***positive ion density***.

The ionosphere is a complex region having diurnal, latitudinal, seasonal and sunspot cycle variations and consequently requires numerical modelling for any meaningful theoretical investigation. A collection of ionosphere models is found in [1]. This paper illustrates how an ionosphere problem of limited scope and duration can be amenable to ***exact solutions in closed forms***. Specifically, we analyze the ***nocturnal decay of the ionosphere above Arecibo***, Puerto Rico during ***sunspot minimum equinox conditions***. Arecibo (latitude 18.46^{o}N) is chosen because it is located in a mid-latitude region where the ionosphere is more stable than either the equatorial or polar regions. Also, during sunspot minimum, the atmosphere and ionosphere are the least turbulent. Further, during equinox conditions, there is least ***inter-hemispheric transport***.

Model Nocturnal Variations of NmF2 and HmF2:

The model theoretical diurnal variations of NmF2 and HmF2 at Arecibo under the above conditions are shown in Fig. 1 and 2 respectively (from [2]). In that

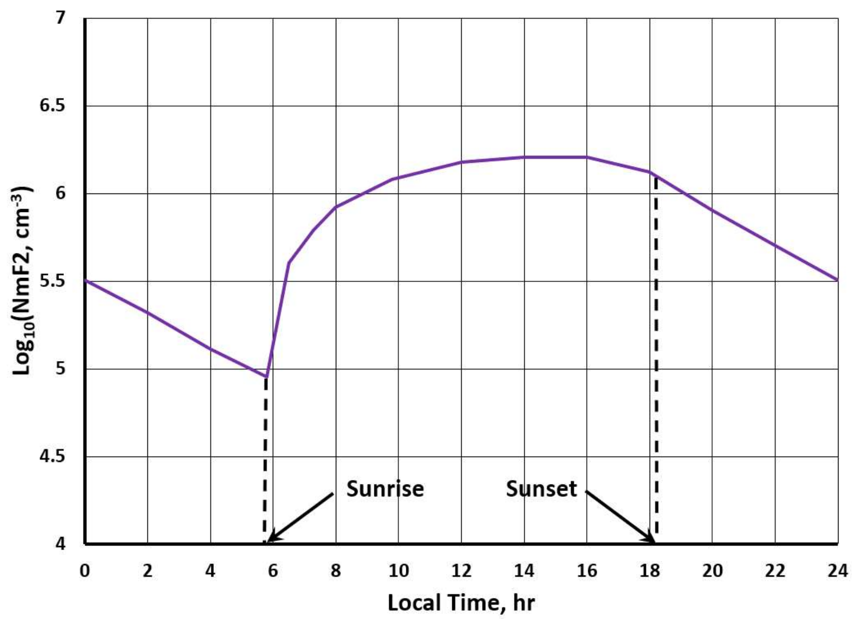
7
6.5
6
5.5
5
4.5
4
Log10(NmF2, cm-3)
0 2 4 6 8 10 12 14 16 18 20 22 24
Local Time, hr
Sunrise
Sunset

Fig. 1

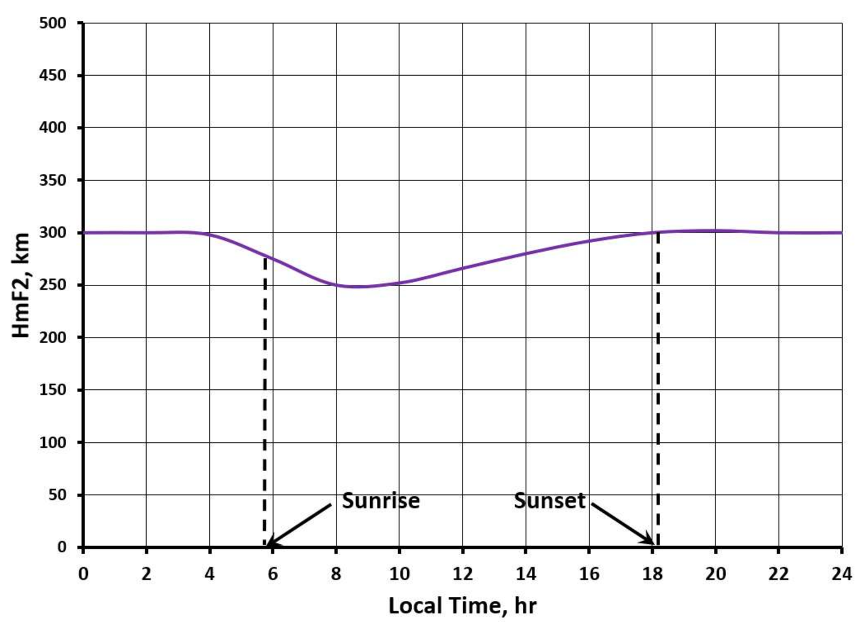
500
450
400
350
300
250
200
150
100
50
0
HmF2, km
0 2 4 6 8 10 12 14 16 18 20 22 24
Local Time, hr
Sunrise
Sunset

Fig. 2

model, ***the neutral atmospheric model*** used was that of Jacchia [3] and Walker [4]. For the low-sunspot condition, an exospheric temperature T_∞ = 800 K was used [5]. For equinox conditions, ***sunrise*** at F2 peak altitudes above Arecibo occurs about 12 min before 6 am. This is exhibited by the steep rise of photoionization occurring at the F2 peak in Fig. 1 from that time. By symmetry, ***sunset*** is assumed to commence 12 min after 18 LT (Fig. 1). The night is therefore 24 min shorter than 12 hr. Figure 2 shows that HmF2, the height of the F2 peak remains constant at 300 km during most of the night, declining gently after 4 am LT. This indicates that the vertical ***electromagnetic drift*** was limited.

Figure 3 shows the model nocturnal decay of the F2 peak over Arecibo under sunspot minimum equinox conditions. What appears as a linear decrease in the ***logarithmic plot*** of Fig. 1 is actually an ***exponentially diminishing pattern*** in Fig. 3. Active during the decay of NmF2 are two distinct loss processes in the ionization:

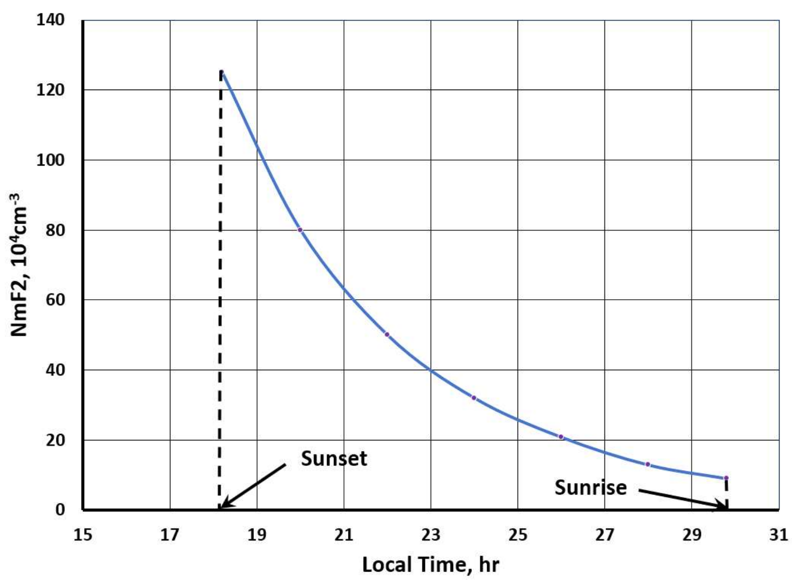

Fig. 3

(1) The quadratic ***ion-electron recombination reaction***:

$$O^+ + e^- \rightarrow O + hv \tag{1}$$

and (2) The linear ***ion-neutral charge transfer reaction***:

$$O^{+} + N_2 \rightarrow N + NO^{+} \tag{2}$$

It is well-known that at HmF2 altitudes, the latter process far dominates the former.

Exact Solutions for Nocturnal Decay of NmF2:

In this section, we derive exact solutions for the nighttime ionospheric decay in three special cases.

(1) Quadratic Loss Rate:

If the ionosphere decays due to ion-electron recombination only, the charge-neutrality condition applies and the ***continuity equation*** for the ion density n is:

$$\frac{dn}{dt} = -\alpha n^2 \tag{3}$$

where α is the quadratic loss coefficient. First, we ***separate the variables and integrate both sides*** from time t_0 to t:

$$\int_{n_0}^{n} \frac{dn}{n^2} = -\alpha \int_{t_0}^{t} dt \tag{4}$$

Upon carrying out the integrations and simplifying, one obtains the peak ion density as a function of time:

$$n_q = \frac{n_0}{1 + \alpha n_0 (t - t_0)} \tag{5}$$

(2) Linear Loss Rate:

If the ionosphere decays according to the ion-neutral charge transfer process only, then the continuity equation for the ion density n is:

$$\frac{dn}{dt} = -\beta n \tag{6}$$

where β is the quadratic loss coefficient. Separating the variables and integrating both sides from time t_0 to t, we get:

$$\int_{n_0}^{n} \frac{dn}{n} = -\beta \int_{t_0}^{t} dt \tag{7}$$

Carrying out the integrations and simplifying, one obtains the peak ion density as a function of time:

$$n_l = n_0 e^{-\beta(t-t_0)} \tag{8}$$

(3) Combined Quadratic + Linear Loss Rates:

In the general case where both the quadratic and linear loss terms are present, the governing equation for the ion density is:

$$\frac{dn}{dt} = -\alpha n^2 - \beta n \tag{9}$$

Separating the variables and integrating both sides from time t_0 to t, we get:

$$\int_{n_0}^{n} \frac{dn}{\alpha n^2 + \beta n} = -\int_{t_0}^{t} dt \tag{10}$$

Carrying out the integrations and simplifying, one obtains the peak ion density as a function of time:

$$n_{q+l} = \frac{\beta}{\left(\alpha + \frac{\beta}{n_0}\right) e^{\beta(t-t_0)} - \alpha} \tag{11}$$

It can be shown that Eq. (11) reduces to Eq. (5) when $\beta = 0$ or to Eq. (8) when $\alpha = 0$.

Results and Discussion:

Mathematically, it is quite cumbersome to utilize Eq. (11) to study this problem. Instead, we analyze Eqs. (5) and (8) to find the pattern of the F2 peak decay under the actions of the quadratic and linear losses separately. Specifically, we solve the inverse problem of determining the constants α and β for the F2 peak ion density to diminish from $n_0 = 1.25 \times 10^6$ cm^{-3} to $n = 9.0 \times 10^4$ cm^{-3} from 18.2 LT to 5.8 LT or $t_0 = 0$ hr to $t = 11.6$ hr. We get from Eqs. (5) and (8): $\alpha = 8.89 \times 10^{-7}$ and $\beta = 0.2268$.

Figure 4 shows the nocturnal decay curves of the peak F2 ion densities as calculated by the quadratic and linear loss terms acting separately. Also shown in the figure is the model nocturnal decay curve with the quadratic and linear losses acting together. Interestingly the two former curves lie on the opposite sides of the model curve. Also, the linear loss curve is very close to the model curve, indicating the fact that ***the linear loss process by ion-neutral charge transfer is the dominant loss process at peak ionospheric heights***. Initially, the quadratic loss curve is a lot steeper than the linear loss curve, but as time progresses, its loss rate becomes slower. This observation can be explained as follows.

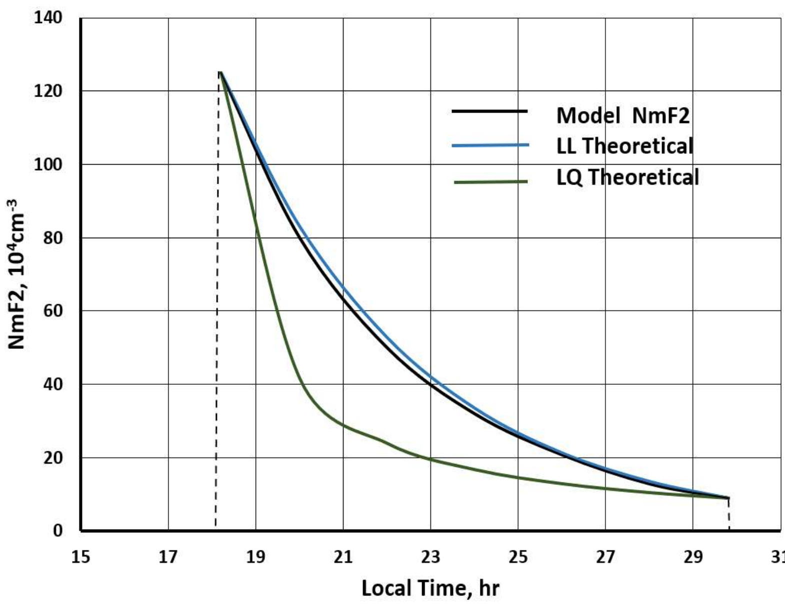

Fig. 4

By ***binomial expansion*** of Eq. (5) and retaining terms up to the second order, we get:

$$n_q = n_0[1 - \alpha n_0(t - t_0) + \alpha^2 n_0^2 (t - t_0)^2] \quad (12)$$

Similarly, by ***series expansion*** of Eq. (8) and retaining terms up to the second order, one has:

$$n_l = n_0[1 - \beta(t - t_0) + \frac{1}{2}\beta^2 (t - t_0)^2] \quad (13)$$

Since $\alpha n_0 > \beta$, the first-order terms dictate that ***n_q decays faster than n_l initially.*** But as $t - t_0$ progresses, the second-order terms ***reverse the rate of decrease in n_q to below that of n_l.***

With the linear loss scheme predicting the nighttime ion densities of the F2 peak only slightly above the model ion densities and the quadratic loss scheme predicting the same densities greatly below the model values, one is naturally tempted to devise a scheme to construct a combination loss scheme comprising a major linear loss process and a minor quadratic loss process. In principle, this would be a ***trial and error method***. With the calculated results obtained in mind, one may well start with a scheme consisting of 90% linear loss and a 10% quadratic loss. The results are in an exceedingly close agreement of ion densities with the model values. This is exhibited in Fig. 5, where the two lines are literally intertwined with one another. In fact, the line thicknesses were reduced to show the separation between the calculated and model density lines. In conclusion, we state that ***the ion-neutral charge-transfer is the dominant loss process, nine times more effective that the ion-electron recombination process***.

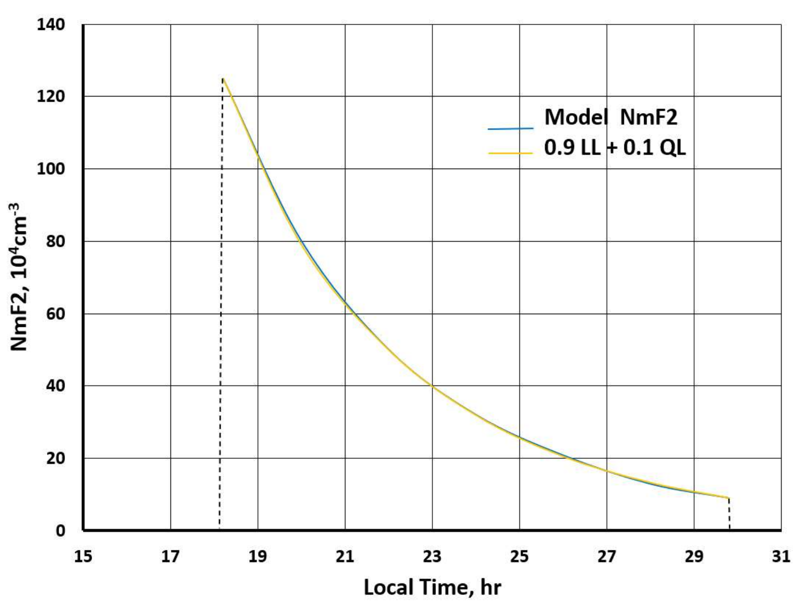

Fig. 5

We should also state that there exists a secondary ion-neutral charge-transfer reaction in the ionosphere, which was ignored in this study:

$$O^+ + O_2 \rightarrow O + O_2^+ \tag{14}$$

This was due to that fact that its inclusion would have complicated the analysis, and nothing new could have been expected.

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