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## **A Mathematical Analysis of Functions Mapping Complex Numbers to Fuzzy Sets: Properties and Applications**

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### **Abstract:**

*In this study, a thorough mathematical framework for analysing functions that translate complex numbers to fuzzy sets is presented. This framework represents a unique function of complex analysis and fuzzy set theory. These kinds of functions are naturally produced in a variety of applications, such as quantum physics, control theory, and signal processing, where there is a coexistence of uncertainty and complex-valued systems. First, we will formally define fuzzy-valued functions over the complex domain, and then we will investigate the essential features of these functions, such as continuity, differentiability, and integrability. Within the scope of this discussion, the idea of fuzzy boundaries is presented and expanded upon in order to characterise convergence. In addition to this, we investigate the structure of level sets and  $\alpha$ -cuts that are linked to fuzzy pictures of complicated functions. Our objective is to determine the role that these*

**[40]**

*structures play in the process of visualising and comprehending fuzzy behaviour. For the purpose of generalising classical ideas in complex function theory to the fuzzy situation, analytical methods such as the extension principle and Zadeh's representation theorem are utilised. There are examples supplied to illustrate the practical ramifications and behaviour of various mappings, and these examples are offered as illustrations. Through the completion of this study, a significant gap between fuzzy mathematics and complex variable theory is bridged. This work provides the framework for additional studies on the stability, approximation, and computing approaches linked to fuzzy-valued complex functions.*

**Keywords:** Mathematical, Mapping, Fuzzy Sets

### **Introduction:**

In contemporary mathematical modelling, the interaction between uncertainty and complex-valued systems is a topic that is gaining growing relevance. This is particularly true in domains such as quantum physics, electrical engineering, and complex dynamical systems. Despite the fact that many issues in the actual world display vagueness or imprecision that cannot be effectively characterised probabilistically, traditional approaches to dealing with uncertainty rely largely on probability theory. When applied in situations like these, fuzzy set theory, which was first presented by Lotfi A. Zadeh in 2015, offers an alternative that is both strong and versatile. The mathematical representation of notions that are fundamentally ambiguous is made possible by it, and it allows for partial membership rather than binary categorisation. Complex analysis has been a major pillar in both pure and practical mathematics throughout the course of its evolution, which has occurred concurrently with the development of fuzzy mathematics. Because of its sophistication and depth in defining oscillatory behaviour, growth features, and transformations, it is a vital tool in a wide variety of fields, including signal processing and fluid dynamics. In spite of this, the unification of fuzzy set theory and complex analysis is still relatively underexplored, particularly with regard to the rigorous study of functions that transfer complex numbers to fuzzy sets. One of the key reasons for writing this paper is to fill up this imaginary empty space. We present a mathematical analysis of functions that take complex numbers as inputs and return fuzzy sets as outputs. These functions are referred to as complex-to-fuzzy mappings, and we recommend that this

analysis be performed. It is necessary to develop new definitions, tools, and interpretations in order to accommodate these mappings, which expand the concept of real-valued fuzzy functions to the complex domain. In the course of their research, they are attempting to find a way to reconcile the exact structure of complex numbers with the approximate and gradational character of fuzzy sets. In the beginning of this work, a formal basis is established for fuzzy sets, fuzzy integers, and fuzzy-valued functions, with a particular focus on complex domains. In the following step, we study fundamental analytical features in this fuzzy-complex situation. These qualities include continuity, differentiability, and integration. In order to define and analyse these qualities, we utilise the Zadeh's extension principle and the level set ( $\alpha$ -cut) decomposition. This is due to the fact that fuzzy sets are not single-valued. In this analysis, one of the most important aspects is to investigate the ways in which standard conclusions in complex analysis, such as the Cauchy-Riemann equations, analyticity, and contour integration, may be generalised or reinterpreted when the codomain is a fuzzy set. As a result of the absence of a linear structure in the space of fuzzy sets and the dependency of operations on  $\alpha$ -levels, this generalisation is not an easy matter. Additionally, we discuss the difficulties that arise when attempting to define the limits and convergence of such functions. To this end, we present concepts such as fuzzy convergence and fuzzy limits for sequences and functions that are defined in the complex domain. The study of complex-to-fuzzy mappings has practical applications in addition to the academic interest that it now possesses. Having a formal framework that takes into account the complicated structure of the system as well as the uncertainty that is inherent in measurements or modeling assumptions can be beneficial for systems that involve imprecise electrical impedances, uncertain frequency response, or fuzzy wave propagation. In conclusion, the purpose of this study is to establish the framework for a more in-depth mathematical investigation of fuzzy functions dealing with complex values. Our goal is to provide new tools and insights for both theoretical mathematics and real-world applications that involve complex uncertainty. This will be accomplished by extending the classical theory of complex functions into the fuzzy domain.

### **1. Literature Review:**

The study of fuzzy-valued functions has been a developing field of mathematical research ever since Lotfi A. Zadeh introduced fuzzy set theory in

2015. This field of study has been under constant development. The work that he did laid the groundwork for the idea of sets with progressive membership, which offered a new mathematical framework for dealing with data that was imprecise and ambiguous. There have been several applications of fuzzy set theory in the field of mathematics over the course of several decades. Some of these applications include fuzzy arithmetic, fuzzy topology, and fuzzy differential equations. On the other hand, the practical application of these concepts within the realm of complex function theory is yet largely unexplored.

Puri and Ralescu (2013), During the early stages of research on fuzzy functions, the primary focus was on mappings from the real numbers  $R$  to fuzzy sets. Through the use of the level set (or  $\alpha$ -cut) representation, the idea of fuzzy-valued functions was introduced, and an approach to fuzzy continuity and differentiability was established. They set the foundations for many of the core concepts and attributes that are still used today, and their work was important in it. The paradigm was further refined by Goetschel and Voxman (2016), who also addressed the concept of fuzzy limits and fuzzy integrals. Their work provided insights into how classical calculus may be extended into the fuzzy domain.

As a result of these concepts, a number of academics started looking into fuzzy differential equations (for example, Kaleva, 2017; Buckley and Feuring, 2000). In these equations, functions with fuzzy values play a significant role in modelling dynamic systems that have unknown parameters. The establishment of existence and uniqueness theorems, the development of fuzzy analogues of classical solution techniques, and the investigation of practical applications in engineering and economics were the primary focusses of these contributions.

The field of complex-valued fuzzy functions, on the other hand, is one that has witnessed comparatively little development. The vast majority of the work that has been done up until now has not regarded complex numbers as constituents of the domain, but rather as parameters in fuzzy-valued functions. When it comes to describing operations and relations in the complex fuzzy domain, the idea of fuzzy complex numbers, which was initially presented by scholars such as Dubois and Prade, serves as a basic step. Following this, subsequent studies made an effort to formalise fuzzy numbers in the complex plane, with a particular emphasis on fuzzy modulus, argument, and polar representations.

Recent studies by Seikkala (2004) and Stefanini (2009), It was possible to extend fuzzy calculus by introducing generalised Hukuhara differentiability, which made it possible for fuzzy-valued functions to have a wider range of applications. Despite the fact that their frameworks were primarily aimed at actual domains, they made it possible to expand fuzzy calculus into more sophisticated contexts. Initial efforts have been done in this area by academics such as Allahviranloo and colleagues (2013), who have investigated fuzzy complex functions, namely those functions that include analytic characteristics and fuzzy complex mappings.

Recent advancements have focused on the visualisation and geometric interpretation of fuzzy mappings in the complex plane. This is often accomplished by the utilisation of level sets or  $\alpha$ -cut representations most of the time. These works shed insight on the non-trivial topology of fuzzy pictures and highlight the difficulties associated with defining concepts such as holomorphicity, conformality, and harmonicity within the setting of fuzzy images.

In order to extend crisp functions to the fuzzy domain, the extension principle remains to be an essential instrument that is utilised during this process. The acceptance of fuzzy arguments has been extended to traditional complex functions (such as exponential, logarithmic, and trigonometric functions) as a result of this concept. The analytical rigour of these transformations continues to be an active study topic.

In spite of the advances described above, there is still a large gap in the formalisation of the theory of functions from complex numbers to fuzzy sets, notably with regard to convergence, differentiability, and integration. In addition, there is a dearth of published material that discusses the algebraic and topological structure of such mappings, as well as their applicability in complex-valued systems that are subject to uncertainty.

In conclusion, although the intersection of fuzzy set theory and complex analysis has the potential to provide exciting results, the majority of the work that has been done thus far has either concentrated on fuzzy-valued real functions or dealt with fuzzy numbers within their own context. A thorough mathematical analysis of functions that transfer complex numbers to fuzzy sets is still a developing area that has a lot of room for theoretical growth and practical application. The purpose of this

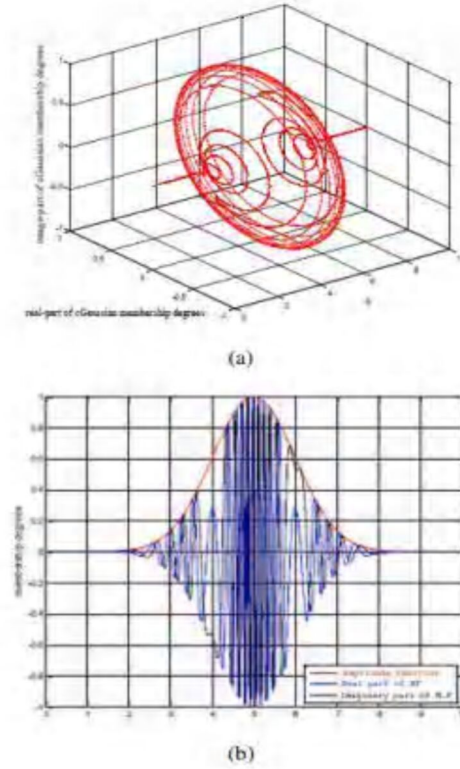
study is to contribute to the narrowing of that gap by developing a systematic framework for analysing such functions. This framework will serve as a foundation for future research in fuzzy complex interpretation.

## 2. Introducing Complex Fuzzy Sets:

In the theory of complex fuzzy sets (CFS), there is the potential for a fresh viewpoint to be offered regarding the study and application of fuzzy systems. CFSs are distinguished from fuzzy complex numbers, which are defined in, by virtue of the fact that they are members of the complex-valued category. It is necessary to take into consideration the amplitude function and the phase function in order to establish a CFS. These two functions, when combined, constitute the membership function. In contrast to the real-valued unit interval space, which is one-dimensional, the complex-valued unit disc space, which is two-dimensional, is where the membership of a CFS is placed. When compared to the membership descriptions of typical fuzzy sets, CFS membership descriptions are able to be far more flexible. Please take into consideration the following: Presented before us is a complicated fuzzy set  $S$ , which has the membership function defined as follows:

$$\begin{aligned}\mu_r(h) &= r_S(h) e^{j\omega_S(h)} \\ &= \text{Re}(\mu_r(h)) + j \text{Im}(\mu_r(h)) \\ &= r_S(h) \cos(\omega_S(h)) + j r_S(h) \sin(\omega_S(h))\end{aligned}\tag{2.1}$$

as  $j$  equals the square root of  $-1$ . The complicated fuzzy set relies on the variable  $h$ . The amplitude function of the complex membership is denoted as  $r_S(h)$ , while the phase function is  $\omega_S(h)$ . It is easy to observe the characteristic of sinusoidal waves in the creation of complex fuzzy functions. A conventional fuzzy set is considered a specific type of complex fuzzy set where  $\omega_S(h)$  is equal to zero. An intricate fuzzy set of the novel Gaussian type is constructed in this work, as seen in Figure 1. There is another example in the piece as well.



**Figure 1: Sample of complicated fuzzy sets of the Gaussian type.**

Analysis of the base variable in relation to the amplitude and imaginary-part memberships.

The following is the design of the complex fuzzy set  $c$ , which is an acronym for the Gaussian type  $(h, m, \sigma, \lambda)$ :

$$\text{Gaussian}(h, m, \sigma, \lambda) = r_S(h, m, \sigma) e^{(j\omega_S(h, m, \sigma, \lambda))} \quad (2.2a)$$

$$r_S(h, m, \sigma) = \text{Gaussian}(h, m, \sigma) = \exp[-0.5((h - m)/\sigma)^2] \quad (2.2b)$$

$$\omega_S(h, m, \sigma, \lambda) = [-0.5((h - m)/\sigma)^2] * ((h - m)/\sigma^2) * \lambda \quad (2.2c)$$

The base variable  $h$  is used in equations (2.2a) to (2.2c) together with  $\{m, \sigma, \lambda\}$ . The parameters that make up the complex fuzzy set are the spread, mean, and phase frequency factor.

### 3. Operations of Complex Fuzzy Sets:

According to traditional fuzzy sets, a function  $\mu$  determines the union of two fuzzy sets,  $A$  and  $B$ , on the set  $U$ , denoted as  $A \cup B$   $\mu : [0, 1] \times [0, 1] \rightarrow [0, 1]$

A membership function  $\mu_{A \cup B}(x)$  might be described in terms of the following:

(i) Standard Union

$$\mu_{A \cup B}(x) = \max[\mu_A(x), \mu_B(x)] \quad (3.1)$$

(ii) Algebraic Sum

$$\mu_{A \cup B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x) \quad (3.2)$$

(iii) Bounded Sum

$$\mu_{A \cup B}(x) = \min[1, \mu_A(x) + \mu_B(x)] \quad (3.3)$$

In order to be equal to the  $T$ -conorms that have been proposed, the fuzzy union must be able to meet the notion of a complex fuzzy set.

#### Complex Fuzzy Union:

Interconnected axioms characterise the intricate fuzzy union. Intuitively acceptable values can only be obtained by satisfying the characteristics of complex fuzzy union functions, which are represented by these axioms.

(a) In this case, the closure property is not satisfied by the complex fuzzy union.

In the algebraic sum, the union function does not satisfy the closure requirement for complex fuzzy sets. This is something that can be demonstrated by the example that is provided below.

**Example 1:** Consider

$$\text{If } \mu_A(x) = \mu_B(x) = j.$$

Then:

$$\mu_A(x) + \mu_B(x) - \mu_A(x) \mu_B(x) + 2j + 1$$

$$\Rightarrow \sqrt{(2^2 + 1^2)} = \sqrt{5} \text{ lies outside the unit circle.}$$

(b) Fuzzy set unions with complicated parameters must be monotone.

The axiomatic monotonicity criterion of classical fuzzy union is usually irrelevant to complex valued membership functions due to the non-linear ordering of complex numbers. Similar to how traditional fuzzy union functions cannot be used with complex-valued membership grades, the max. and min. operators cannot be used with these grades either. We may apply the standard fuzzy definition of union here while using the same strategy for the complex component. Here we see one how an alternate method of describing phases may look.

**Definition:** Let  $A$  and  $B$  be two complex fuzzy sets on  $U$ , with membership functions  $\mu_A(x)$  and  $\mu_B(x)$ , respectively, for the sake of argument. It is possible to define the complicated fuzzy union  $A \cup B$  as

$$\mu_{A \cup B}(x) = [r_A^-(x) \otimes r_B^-(x)] e^{j\omega_{\{A \cup B\}}(x)} \quad (3.4)$$

Where  $\oplus$  represents the  $T$ -conorm and  $\omega_{A \cup B}$  is defined as follows:

(a) (Sum)

$$\omega_{A \cup B} = \omega_A + \omega_B \quad (3.5)$$

(b) (Max)

$$\omega_{A \cup B} = \max(\omega_A, \omega_B) \quad (3.6)$$

(c) (Min)

$$\omega_{A \cup B} = \min(\omega_A, \omega_B) \quad (3.7)$$

(d) (Winner takes all)

$$\omega_{A \cup B} = \begin{cases} \omega_A, & \text{if } r_A > r_B \\ \omega_B, & \text{if } r_B \geq r_A \end{cases} \quad (3.8)$$

**Complex Fuzzy Intersection:**

The following definition makes it abundantly evident that the derivation of fuzzy complex intersection is strongly connected to complex fuzzy union. This is something that should be taken into consideration.

**Definition:** Two complicated fuzzy sets,  $A$  and  $B$ , defined on  $U$ , intersect at the given point according to the following definition:

$$\mu_{\{A \cap B\}}(x) = [r_A^-(x) \otimes r_B^-(x)] e^{\wedge \{j\omega_{\{A \cap B\}}(x)\}} \quad (3.9)$$

In this context, the symbol  $*$  represents any  $t$ -norm function, which refers to functions that fulfil all the axiomatic conditions of classical fuzzy intersection. Such functions include, for instance, the algebraic product and the minimum. It is the specific form that  $\omega_{A \cap B}(x)$  is dependent upon the application under consideration.

Possible methods of calculating  $\omega_{A \cap B}(x)$  are given in (3.5)-(3.8). Note that in theory  $\omega_{A \cap B}(x)$  and  $\omega_{A \cup B}(x)$  may concurrently be specified by the same function (for example, the minimum value). In actuality, however, it is highly improbable that this approach will prove to be an effective way for computing these terms.

**Complex Fuzzy Complement:**

The complement of a typical fuzzy set is defined by Klir and Yuan using an axiomatic approach. This definition can be found in their work. The criteria of the axiomatic framework can be satisfied by a number of alternative complement functions.

Some examples are as follows:

(1) The standard complement:

$$c(\mu_f^-(x)) = 1 - \mu_f^-(x); \mu_f^-(x) \in [0, 1] \quad (3.10)$$

The vast majority of fuzzy applications have made use of this particular version of complement, which meets all four of the axiomatic conditions.

(2) Step-threshold complement:

$$c(\mu_f^-(x)) = \{1, \text{ for } \mu_f^-(x) \leq r, 0, \text{ for } \mu_f^-(x) > r, \mu_f^-(x) \in [0, 1] \quad (3.11)$$

One and only the axiomatic skeleton is satisfied by this particular kind of complement (Axioms 1 and 2).

(3) Enhancements to the Sugeno class:

$$c_\lambda(\mu_f^-(x)) = (1 - \mu_f^-(x)) / (1 + \lambda \mu_f^-(x)), \mu_f^-(x) \in [0, 1], \lambda \in (-1, \infty) \quad (3.12)$$

### **Complex fuzzy complement:**

When attempting to determine the complement of a complicated fuzzy set, it would appear to be the most obvious course of action to utilise the same axiomatic concept and the complement functions that are derived from it. Unfortunately, this strategy is fraught with a number of challenges. There is a possibility that more favourable outcomes will be accomplished if each component, both actual and fictional, is taken into consideration separately.

**Definition:** Let  $\mu_S(x)$  denote the complex membership grade contained inside  $S$  and suppose that  $S$  is a complex fuzzy set on  $U$ , the universe of discourse. Notation  $c(s)$  will be used to represent the complex fuzzy complement of  $S$  of type  $c$ , which is specified by the function.

$$c : \{a \in C, a \leq 1\} \rightarrow \{b \in C, b \leq 1\}$$

Which assigns a value  $c(\mu_S(x))$  to all  $x$  in  $U$ .

The complex fuzzy complement function must satisfy certain axiomatic criteria, the most basic of which are as follows:

- **Axiom 1 (amplitude boundary conditions):**

$$|a| = 0 \Rightarrow |c(a)| \quad \text{and} \quad |a| = 1 \Rightarrow |c(a)| = 0$$

- **Axiom 2 (amplitude monotonicity):**

$$a, b \in C, \text{ with } |a|, |b| \in [0, 1], \text{ if } a \leq b, \text{ then } |c(a)| \geq |c(b)|$$

Keeping  $x$ 's membership phase in the complement set, denoted by  $(x)$ ,  $\mu_S$ , was chosen as the appropriate approach for the sake of computation, which would be technically correct. This view of the outcome, however, is incorrect as it takes for granted that the membership phase is significantly affected by the set-theoretic operation of complement. No, that's not right. Note that complementation is strictly confined to the amplitude term of complex membership grades and has nothing to do with the membership phase. It is important to highlight this.

### Features of $\xi$ -Complex Fuzzy Subgroups from an Algebraic Perspective:

Here we provide a new concept of  $\xi$ -CFSG defined on  $\xi$ -CFS and prove some basic algebraic properties of this phenomenon.

**Definition:** A  $\xi$ -CFS that is homogeneous  $A$   $\xi$ -complex fuzzy subgroup ( $\xi$ -CFSG) is a subgroup  $A\xi$  of a group  $G$  that admits the following requirements:

1.  $\mu_A(mn) \geq \min[\mu_A(m), \mu_A(n)]$
2.  $\mu_A(m^{-1}) \geq \mu_A(m), \forall m, n \in G$

All  $\xi$ -CFSGs defined on group  $G$  are represented by their family  $F\xi(G)$ .

**Proposition 1:** Each  $\xi$ -CFSG  $(G) A^\xi$  satisfies the following properties:

3.  $\mu_A(m) \leq \mu_A(e), \forall m \in G$
4.  $\mu_A(mn^{-1}) = \mu_A(nm^{-1}) \Rightarrow \mu_A(m) = \mu_A(n), \forall m, n \in G$

**Proof:**

(1) Let  $m \in G$ , then

$$\begin{aligned} \mu_{\{A^\wedge\{\xi\}\}}^-(e) &= \mu_{\{A^\wedge\{\xi\}\}}^-(mm^{-1}) \\ &\geq \min\{\mu_{\{A^\wedge\{\xi\}\}}^-(m), \mu_{\{A^\wedge\{\xi\}\}}^-(m^{-1})\} \\ &= \mu_{\{A^\wedge\{\xi\}\}}^-(m). \end{aligned}$$

(2) Let  $m, n \in G$ , then

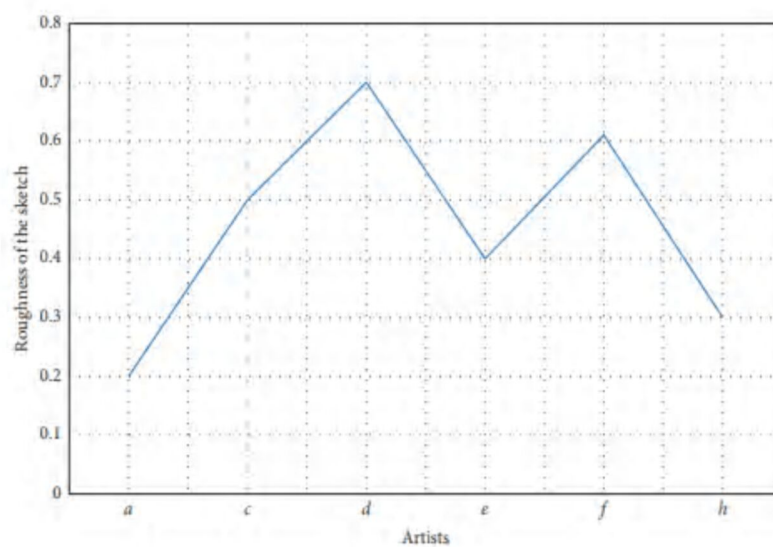
$$\begin{aligned}
\mu_{\{A^{\wedge}\{\xi\}\}}^-(m) &= \mu_{\{A^{\wedge}\{\xi\}\}}^-(mn^{\wedge}\{-1\}) \\
&\geq \text{Min}\{\mu_{\{A^{\wedge}\{\xi\}\}}^-(mn^{\wedge}\{-1\}), \mu_{\{A^{\wedge}\{\xi\}\}}^-(n)\} \\
&= \min\{\mu_{\{A^{\wedge}\{\xi\}\}}^-(e), \mu_{\{A^{\wedge}\{\xi\}\}}^-(n)\} \\
&= \mu_{\{A^{\wedge}\{\xi\}\}}^-(n).
\end{aligned}$$

We examine the circumstances in which a certain CFS is  $\xi$ -CFSG in the subsequent outcome.

**Proposition 2:** Let  $A$  be a CFS( $G$ ), such that  $\mu_A(m-1) \mu_A(m)$ ,  $\forall m \in G$ .

Moreover, Let  $\xi \leq \alpha$ , where  $\alpha = \inf\{\mu_A^-(m) : m \in G\}$ . Then,  $A^{\wedge}\{\xi\}$  is  $\xi$ -CFSG( $G$ ).

**Proof:** We may deduce that  $\mu_A^-(m) = \xi$  by applying the provided requirements to any  $m \in G$ . Using Definition on the inequality given above, we get that  $\mu_{A^{\wedge}\{\xi\}}(m) = \xi$ , therefore,  $\mu_A^-(mn) = \min\{\mu_A^-(mn), \xi\}$  and  $\mu_{\{A^{\wedge}\{\xi\}\}}^-(mn) \geq \min\{\mu_{\{A^{\wedge}\{\xi\}\}}^-(m), \mu_{\{A^{\wedge}\{\xi\}\}}^-(n)\}$ , for all  $m, n \in G$ . Moreover, by using the given condition  $\mu_A^-(m^{\wedge}\{-1\}) = \mu_A^-(m)$ , we get  $\mu_{\{A^{\wedge}\{\xi\}\}}^-(m^{\wedge}\{-1\}) = \mu_{\{A^{\wedge}\{\xi\}\}}^-(m)$ .



**Figure 2:** Table 1 is interpreted graphically in this example.

**Table 1: The performers' performances following the conclusion of the second phase**

| Artists | Performance of the Artists ( $\omega_A^-$ ) |
|---------|---------------------------------------------|
| a       | $0.5\pi$                                    |
| b       | $0.6\pi$                                    |
| c       | $1.5\pi$                                    |
| d       | $0.6\pi$                                    |
| e       | $0.7\pi$                                    |
| h       | $0.4\pi$                                    |

As demonstrated by the subsequent finding, each and every CFSG is invariably  $\xi$ -CFSG.

**Proposition 3:** Every CFSG  $A$  is  $\xi$ -CFSG of a group  $G$ .

**Proof:** Using Definition, we can determine that for any  $m$  and  $n$  that belongs to  $G$ , we have  $\mu_{\{A^{\wedge}\{\xi\}\}}^-(mn) = \min\{\mu_A^-(m), e \text{ application of Definition in the above relation gives us } \mu_A^-(n)\}$ .

Moreover

$$\begin{aligned}\mu_{\{A^{\wedge}\{\xi\}\}}^-(m^{\wedge}\{-1\}) &= \min\{\mu_A^-(m^{\wedge}\{-1\}), \xi\} \\ &\geq \min\{\mu_A^-(m), \xi\}\end{aligned}$$

Hence,  $A$  is a  $\xi$ -CFSG( $G$ ).

**Remark 1:** In most cases, the opposite of what is stated in Proposition 3 is never true. It is possible to see an example of an algebraic a reality concerning the example that follows.

**Example 2:**  $e$  CFS  $A$  defined on a  $G\{1, -1, i, -i\}$  is given as

$$A(m) = [[0.2e^{\{i\pi\}}, 0.4e^{\{i\pi\}}, 0.3e^{\{0.9\pi\}}], [-1, -1, i]]$$

$e\xi$ -CFSG( $G$ ) corresponding to the value  $\xi$

$0.1 ei 0.5\pi$  is given by

$$A^{\wedge\{\xi\}}(m) = [[0.1e^{\{i0.5\pi\}}, 0.1e^{\{i0.5\pi\}}, 0.1e^{\{i0.5\pi\}}], [1, -1, i]]$$

To add insult to injury, definition does not apply to  $A$ , which means that it does not qualify as  $\text{CFSG}(G)$ . It is demonstrated in the following conclusion that the intersection of any two  $\xi$ -CFSG is also a  $\xi$ -CFSG.

**Proposition 4:** For any two  $A^{\wedge\{\xi\}}, B^{\wedge\{\xi\}} \in F^{\wedge\{\xi\}}(G)$ ,  $(A \cap B)^{\wedge\{\xi\}} = A^{\wedge\{\xi\}}$

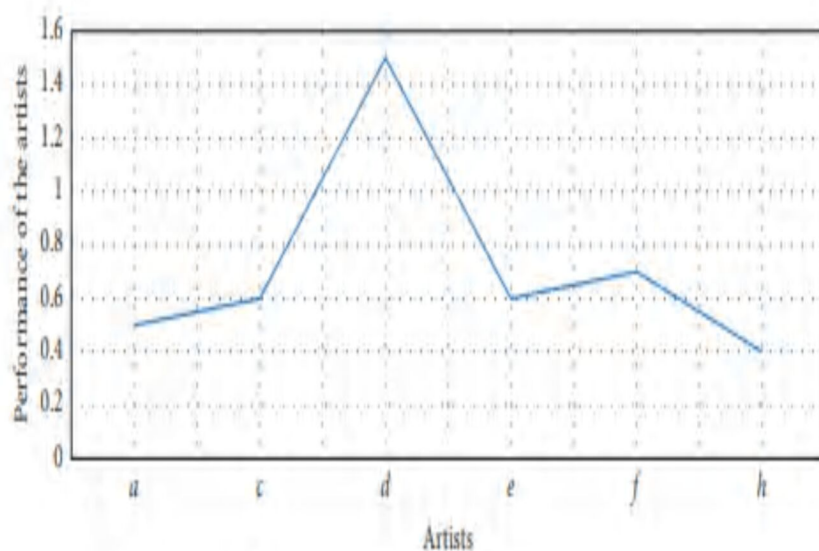
**Proof:** By using Proposition 1, for any two elements  $m, n \in G$ ,

$$\begin{aligned} \mu_{\{(A \cap B)^{\wedge\{\xi\}}\}}^-(mn) &= \mu_{\{A^{\wedge\{\xi\}} \cap B^{\wedge\{\xi\}}\}}^-(mn) \\ &= \min\{\mu_{\{A^{\wedge\{\xi\}}\}}^-(mn), \mu_{\{B^{\wedge\{\xi\}}\}}^-(mn)\} \end{aligned}$$

In the context of the preceding relation, the application of Definition results in that

$$\mu_{\{(A \cap B)^{\wedge\{\xi\}}\}}^-(mn) = \min\{\mu_{\{(A \cap B)^{\wedge\{\xi\}}\}}^-(m), \mu_{\{(A \cap B)^{\wedge\{\xi\}}\}}^-(n)\}$$

Moreover,



**Figure 3: An illustration of the interpretation**

$$\begin{aligned}
\mu_{\{(A \cap B)^{\wedge \{\xi\}}\}}^-(m^{\wedge \{-1\}}) &= \mu_{\{(A \cap B)^{\wedge \{\xi\}}\}}^-(m^{\wedge \{-1\}}) \\
&\geq \min \{ \mu_{\{A^{\wedge \{\xi\}}\}}^-(m), \mu_{\{B^{\wedge \{\xi\}}\}}^-(m) \} \\
&= \mu_{\{(A \cap B)^{\wedge \{\xi\}}\}}^-(m).
\end{aligned}$$

This concludes the proof.

### Conclusion:

At the intersection of complex analysis and fuzzy set theory, this study has investigated the new and intellectually stimulating field of functions mapping complex numbers to fuzzy sets. We have established the groundwork for comprehending such mappings by building a strict mathematical framework, which starts with the exact definitions of fuzzy sets, fuzzy integers, and fuzzy-valued functions over the complex domain. Using level set ( $\alpha$ -cut) analysis and the extension principle extensively, we investigated important analytical properties-including continuity, differentiability, and integrability-in this hybrid setting. Using these methods, we were able to successfully transfer classical ideas from complex function theory to the fuzzy domain, but we also saw how fuzzy sets differ significantly and the difficulties that come with them. In order to account for the characteristics of imprecision and partial membership, our examination of fuzzy limits, convergence, and complex differentiability uncovered that although some similarities to classical complex analysis do exist, they need meticulous reformulation. Furthermore, the work shed light on the complex plane's complicated geometric and topological behaviour of fuzzy mappings. Areas that require the modelling of uncertainty in complex-valued systems, like quantum computing, electrical engineering, control systems, and uncertain signal processing, can benefit from fuzzy images and their associated level sets due to their visual and structural complexity. This paper proves, among other things, that fuzzy-valued complex functions are useful instruments with theoretical and practical depth, rather than just abstract generalisations. We provide the groundwork for future research on more complicated behaviours, such as analyticity, harmonicity, and fuzzy boundary value issues, by expanding basic mathematical frameworks. We still have a long way to go, despite the advances. Theorems such as Cauchy's integral theorem and residue theorem can be extended to the fuzzy domain, and there is still room for research into developing numerical methods for computing fuzzy-valued complex functions, as well as investigating functional analysis of fuzzy Hilbert spaces with complex arguments.

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