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Ramanujan Number is an Absolute Pseudoprime

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Abstract:

Ramanujan was a great mathematician and versatile geniuses. He had a keen interest to solve the problems related to mathematics. In this way, solved so many problems which was unsolved. Ramanujan number is 1729. There is a history behind this number.

History:

Ramanujan was born in Erode, a small village South-West of Madras in 1887. In 1912, he was in a job in Madras. G.H. Hardy habitant of Cambridge, was a good friend of Ramanujan. His full name was Sriniwas Ramanujan. In April 1914, Ramanujan was with G.H. Hardy in Cambridge and there they solved so many unsolved problems. After sometime, Ramanujan thought himself, India is my country, so he had a great desire to come India and wrote a letter to Madras University. But no Indian took interest about his returning. In 1917, he fell incrubly ill, then G.H. Hardy admitted that fellow in a big hospital for better treatment.

One day Hardy came to meet him with his own car and said how are you, my friend. Ramanujan replied, my friend Hardy, you are very lucky, How I am lucky. Hardy said, then Ramanujan replied, your car number is 1729 and this number is the sum of the cubes of two numbers (integers)

$$(12)^3 + 1^3 = 1729$$

Also $9^3 + 10^3 = 1729.$

Hardy said, I never thought about this number. In fact, you are a great mathematician. Hardy said, your health is decreasing day by day and how did you do this calculation, beyond my imagination. At the age of 33 yrs. (in 1920). He want for ever, therefore 1729 is regarded as Ramanujan number. In this time he is no more. But Ramanujan's activities are present here.

Now to show this number is an absolute pseudoprime. For this, we give some definitions. Which is quite necessary.

Prime number:

An integer $p > 1$ is called a prime number if it is divisible by itself and unity as 2, 3, 5, 7, 11, 13, 17,.... in which 2 is only even prime and all the primes are odd.

An integer > 1 is called a composite number, if it is not a prime as 4, 6, 8, 9, 10, 12,....

Theorem 1: If p is a prime and a is any integer, then either $(a, p) = 1$ or $(a, p) = p$ i.e. a is a multiple of p.

Proof: As p is a prime, then it has two divisors 1 and p, therefore $(a, p) = 1$ or $(a, p) = p$ if $(a, p) = 1$ then our theorem is proved. If $(a, p) = p$ then obviously a is a multiple of p.

Congruency:

If we write $a \equiv b \pmod{n}$, this is only possible, when $a - b$ is divisible by n.

If it is not divisible by n, then we write

$$a \not\equiv b \pmod{n}$$

Fermat's little theorem:

If p is a prime and $(a, p) = 1$ then $a^{p-1} \equiv 1 \pmod{p}$. But the converse of this theorem is not true. We shall show it by means of an example.

For some integer a and p, p is not a prime as

$$2^{340} \equiv 1 \pmod{341} \text{ or}$$

$$2^{341-1} \equiv 2 \pmod{341}$$

But 341 is not a prime.

Pseudoprime:

An integer n is called a pseudoprime if $2^n \equiv 2 \pmod{n}$. Here the number 341 is a pseudoprime as it can be written as

$$2^{341} \equiv 2 \pmod{341}.$$

Absolute Pseudoprime:

A composite number n is called an absolute pseudoprime if $a^n \equiv a \pmod{n}$ $\forall$ integers a. The least absolute pseudoprime is 561.

Now to show 561 is an absolute pseudoprime.

Prime factor of 561 = 3.11.17 and if $(a, 561) = 1$, then $(a, 3) = 1$, $(a, 11) = 1$, $(a, 17) = 1$, where 3, 11, 17 all are primes.

Now by Fermat's theorem

$$a^{3-1} \equiv 1 \pmod{3}$$

$$\Rightarrow \quad a^2 \equiv 1 \pmod{3}$$

Similarly $a^{10} \equiv 1 \pmod{11}$ and $a^{16} \equiv 1 \pmod{17}$

$$\Rightarrow \quad a^{560} = (a^2)^{280} \equiv 1 \pmod{3}$$

$$a^{560} = (a^{10})^{66} \equiv 1 \pmod{11}$$

$$a^{560} = (a^{16})^{35} \equiv 1 \pmod{17}$$

$$\Rightarrow \quad a^{560} \equiv 1 \pmod{3.11.17}$$

$$\Rightarrow \quad a^{560} \equiv 1 \pmod{561}$$

$$\Rightarrow \quad a^{561} \equiv a \pmod{561}$$

Proved.

Now our question is Ramanujan number 1729 is also an absolute pseudoprime.

Proof: We can write 1729 = 7.13.19 if $(a, 1729) = 1$

then $(a, 7) = 1$, $(a, 13) = 1$, $(a, 19) = 1$.

By Fermat's theorem

$$a^{7-1} \equiv 1 \pmod{7}$$

$$\Rightarrow \quad a^{6} \equiv 1 \pmod{7}$$

$$a^{12} \equiv 1 \pmod{13}, \quad a^{18} \equiv 1 \pmod{19}$$

$$\Rightarrow \quad a^{1728} = (a^{6})^{288} \equiv 1 \pmod{7}$$

$$a^{1728} = (a^{12})^{144} \equiv 1 \pmod{13}$$

$$a^{1728} = (a^{18})^{96} \equiv 1 \pmod{19}$$

$$\Rightarrow \quad a^{1728} \equiv 1 \pmod{7.13.19}$$

$$\Rightarrow \quad a^{1728}.a \equiv a \pmod{1729}$$

$$\Rightarrow \quad a^{1729} \equiv a \pmod{1729}$$

therefore, 1729 is an absolute pseudoprime.

Proved.

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