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Common Fixed Point Theorem for a (ϕ, ψ)-rational contraction

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Abstract:

Common fixed point theorem for a generalized (ϕ, ψ)-contractive type mapping on a complete 2-metric space.

Keywords: fixed point, common fixed point, 2-metric space, (ϕ, ψ)- contraction

1. Introduction:

In 1963 Gähler[1] introduced a concept of 2-metric spaces as a generalization of metric spaces. The 2-metric space is used to measure the area of triangle in

R^2 as the inspiration example. It has been show by Gähler that in 2-metric d is non-negative. After Gähler many authors obtained results in these spaces [2, 3, 4, 9, 11, 12]. Application of fixed point theory in 2-metric spaces is in medicine, economics, game theory, etc.

Definition 1.1[1]: Let X be a non-empty set and $d : X.X.X \to R$. It for all $x, y, z \in X$ and u in X we have,

I. $d(x, y, z) = 0$. If at least two of x, y, z are equal.

II. For all x, y there exists a point z in X such that $d(x, y, z) \neq 0$.

III. $d(x, y, z) = d(x, z, y) = d(y, x, z) = d(y, z, x) = d(z, x, y) = d(z, y, x)$, for all $x, y, z \in X$.

IV. $d(x, y, z) = d(x, y, u) + d(x, u, z) + d(u, y, z)$, then d is called a 2-metric on X and the pair (X, d) is called to be 2-metric space.

Example 1.1: Let a mapping $d : R^3 \to [0, \infty)$ be defined by $d(x, y, z) = \min\{Ix\text{-}yI, Iy\text{-}zI, Iz\text{-}xI\}$. Then d is a 2-metric on R.

In 2001, Rhoades[5] established a fixed point theorem for $T : X \to X$ in concept of metric space.

Theorem 1.1[5]: Let X be complete metric spaces and let $T : X \to X$ be any mapping. Assume that for every $x, y \in X$,

$$d(Tx, Ty) \leq d(x, y) - \phi(d(x, y)),$$

where, $\phi : [0, \infty) \to [0, \infty)$ is a continuous and non-decreasing function with $\phi(0) = 0$ and $\phi(t) > 0$ for all $t > 0$. Then T has a unique fixed point.

In 2008, Dutta and Choudhary [6] obtained the generalization of theorem (1.1).

Theorem 1.2[6]: Let (X, d) be a complete metric space and let $T : X \to X$ be two mapping. Assume that for every $x, y \in X$.

$$\Psi(d(Tx, Ty)) \leq \Psi(d(x, y)) - \phi(d(x, y)),$$

where,

(i) $\Psi : [0, \infty) \to [0, \infty)$ is a continuous and monotone non-decreasing function with $\Psi(t) = 0$ if and only if $t = 0$.

(ii) $\phi : [0, \infty) \to [0, \infty)$ is a lower-semi continuous function with $\phi(t) = 0$ if and only if $t = 0$. Then T has a unique fixed point.

In 2009, Zhang et al.[7] obtained the following generalization of theorem (1.1).

Theorem 1.3[7]: Let (X, d) be a complete metric space and let $T, S : X \to X$ be two mappings. Assume that for every $x, y \in X$.

$$d(Tx, Sy) \leq M(Tx, Sy) - \phi(M(Tx, Sy)),$$

where $M(Tx, Sy) = \max\{d(x, y), d(x, Tx), d(y, Sy), \frac{d(y, Tx)+d(x, Sy)}{2}\}$ and ϕ is defined as in Theorem (1.1). Then there exists a unique point $z \in X$ such that $z = Tz = Sz$.

In 2009, Dorić [8] obtained common fixed point theorem for two mapping generalizes above results.

Theorem 1.4[8]: Let (X, d) be a complete metric space and let $T, S : X \to X$ be two mappings. Assume that for every $x, y \in X$

$$\Psi(d(Tx, Ty)) \leq \Psi(M(Tx, Ty)) - \phi(M(Tx, Sy)),$$

where Ψ and ϕ defined as in theorem (1.2) and $M(Tx, Sy) = \max\{d(x, y), d(x, Tx),$ $d(y, Sy), \frac{d(y, Tx)+d(x, Sy)}{2}\}$.

Then there exists a unique point $z \in X$ such that

$$z = Tz = Sz.$$

In 2017, Fei He et al.[11] proved the common fixed point theorem for two mapping satisfying a generalized (Ψ, ϕ)- Suzuki weak contractive type condition in a complete metric space.

Theorem 1.5[12]: Let (X, d) be a complete metric space and let $T, S : X \to X$ be two mappings. Assume that for every $x, y \in X$

$$\frac{1}{2}\min\{d(x, Tx), d(y, Sy)\} \le d(x, y) \text{ implies}$$
$$\Psi(d(Tx, Sy)) \le \Psi(M(Tx, Ty)) - \phi(M(Tx, Sy)),$$

where Ψ and ϕ defined as in theorem (1.2) and $M(Tx, Sy) = \max\{d(x, y), d(x, Tx), d(y, Sy), \frac{d(y,Tx)+d(x,Sy)}{2}\}$.

Then there exists a unique point $z \in X$ such that $z = Tz = Sz$.

In 2020, Arya et al.[12] obtained the results for the generalized (Ψ, ϕ)- Suzuki weak contraction under a rational expression.

Theorem 1.6[13]: Let (X, d) be a complete metric space and let $T, S : X \to X$ be two mappings. Assume that for every $x, y \in X$

$$\frac{1}{2}\min\{d(x, Tx), d(y, Sy)\} \le d(x, y) \text{ implies}$$
$$\Psi(d(Tx, Sy)) \le \Psi(N(Tx, Ty)) - \phi(N(Tx, Ty)),$$

where $N(Tx, Ty) = \max\{d(x, y), d(y, Sy)(\frac{1+d(x,Tx)}{1+d(x,y)})\}$ and Ψ and ϕ are defined as in theorem (1.2).

Then there exists a unique point $z \in X$ such that $z = Tz = Sz$.

In 2023, Arya et al.[13] obtained the results for the generalized (Ψ, ϕ)- Suzuki weak contraction under a rational expression.

Theorem 1.7[14]: Let (X, d) be a complete metric space and let $T, S : X \to X$ be two mappings. Assume that for every $x, y \in X$

$$\Psi(d(Tx, Sy)) \le \Psi(M_1(Tx, Sy)) - \phi(M_1(Tx, Sy)),$$

where $M_1(Tx, Ty) = \max\{d(x, y), d(x, Tx), d(y, Sy), \frac{d(y,Tx)+d(x,Sy)}{2}, \frac{d(x,Tx)+d(y,Sy)}{2}, d(y, Sy)(\frac{1+d(x,Tx)}{1+d(x,y)}), d(x, Tx)(\frac{1+d(y,Sy)}{1+d(x,y)})\}$ and Ψ and ϕ are defined as in theorem (1.2). Then there exists a unique fixed point $z \in X$ such that $z = Tz = Sz$.

Definition 1.2[1]: A sequence $\{x_n\}_{n\in N}$ in a 2-metric space (X, d) is said to be a Cauchy sequence if $\lim_{m,n\to\infty} d(x_m, x_n, a) = 0$ for all $a \in X$.

Definition 1.3[1]: A sequence $\{x_n\}_{n\in N}$ in a 2-metric space (X, d) is said to be a convergent at $a \in X$ if $\lim_{m,n\to\infty} d(x_n, x, a) = 0$ for all $a \in X$.

Definition 1.4[1]: A 2- metric space (X, d) is said to be complete if every Cauchy sequence in X is convergent.

2. Main Result:

The purpose of this paper is to study the (Ψ, ϕ)- Suzuki contraction under a rational expression Arya et al.[13] on the setting of 2-metric space.

Theorem 2.1: Let X be a complete 2-metric space and Let $T, S : X \to X$ be two mappings. Assume that for every, $x, y, a \in X$.

$$\Psi(d(Tx, Sy, a)) \leq \Psi(M_1(Tx, Sy, a)) - \phi(M_1(Tx, Sy, a)) \qquad (2.1)$$

Where $M_1(Tx, Ty, a) = \max\{d(x, y, a), d(x, Tx, a), d(y, Sy, a), \frac{d(y,Tx,a)+d(x,Sy,a)}{2},$

$$\frac{d(x,Tx,a)+d(y,Sy,a)}{2}, d(y, Sy, a)\left(\frac{1+d(x,Tx,a)}{1+d(x,y,a)}\right), d(x, Tx, a)\left(\frac{1+d(y,Sy,a)}{1+d(x,y,a)}\right)\}$$

and Ψ and ϕ are defined as in theorem (1.2). Then there exists a unique fixed point $z \in X$ such that $z = Tz = Sz$.

Proof: Suppose x_0 is an arbitrary. Then we can choose $x_1 = Sx_0$, $x_2 = Tx_1$, $x_3 = Sx_2$ and $x_4 = Tx_3$. In general, we can construct a sequence $\{x_n\}_{n\in N}$ in X such that $x_{2n+2} = Tx_{2n+1}$ and $x_{2n+1} = Sx_{2n}$.

Now, if n is odd then, by (2.1) we have,

$$\Psi(d(Tx_n, Sx_{n-1}, a)) \leq \Psi(M_1(Tx_n, Sx_{n-1}, a)) - \phi(M_1(Tx_n, Sx_{n-1}, a))$$

Where $M_1(Tx_n, Sx_{n-1}, a) = \max\{d(x_n, x_{n-1}, a), d(x_n, Tx_n, a), d(x_{n-1}, Sx_{n-1}, a),$
$[d(x_{n-1}, Tx_n, a) + d(x_n, Sx_{n-1}, a)]/2, [d(x_n, Tx_n, a)$
$+ (x_{n-1}, Sx_{n-1}, a)]/2, d(x_{n-1}, Sx_{n-1}, a) [(1 + d(x_n, Tx_n, a))/$
$(1 + d(x_n, x_{n-1}, a))], d(x_n, Tx_n, a) [(1 + d(x_{n-1}, Sx_{n-1}, a))/$
$(1 + d(x_n, x_{n-1}, a))]\}$

$$= \max\{d(x_n, x_{n-1}, a), d(x_n, x_{n+1}, a), d(x_{n-1}, x_n, a), [d(x_{n-1}, x_{n+1}, a) + d(x_n, x_n, a)]/2, [d(x_n, x_{n+1}, a) + d(x_{n-1}, x_n, a)]/2, d(x_{n-1}, x_n, a)[(1 + d(x_n, x_{n+1}, a))/(1 + d(x_n, x_{n-1}, a))], d(x_n, x_{n+1}, a)[(1 + d(x_{n-1}, x_n, a))/(1 + d(x_n, x_{n-1}, a))]\}$$

$$= \max\{d(x_n, x_{n-1}, a), d(x_n, x_{n+1}, a)\}$$

So, we obtain

$$\Psi(d(Tx_n, Sx_{n-1}, a)) \leq \Psi(\max\{d(x_n, x_{n-1}, a), d(x_n, x_{n+1}, a)\}) - \phi(\max\{d(x_n, x_{n-1}, a), d(x_n, x_{n+1}, a)\}) \quad (2.2)$$

If $d(x_n, x_{n+1}, a) > d(x_{n-1}, x_n, a)$ for some n, then (2.2) gives

$$\Psi(d(x_n, x_{n+1}, a)) \leq \Psi(d(x_n, x_{n+1}, a)) - \phi(d(x_n, x_{n+1}, a)) < \Psi(d(x_n, x_{n+1}, a)),$$

which is a contradiction. Hence for all n, we get

$$\Psi(d(x_n, x_{n+1}, a)) \leq \Psi(d(x_{n-1}, x_n, a)) - \phi(d(x_{n-1}, x_n, a)).$$

Consequently, we have

$$\Psi(d(x_n, x_{n+1}, a)) \leq \Psi(d(x_{n-1}, x_n, a)) \quad (2.3)$$

In an analogous way, we can show that condition (2.3) is true for even values of n.

By the property of Ψ, for all $n \in N$, the Positive integers, we have

$$d(x_n, x_{n+1}, a) \leq d(x_{n-1}, x_n, a) \quad (2.4)$$

Moreover, the sequence $\{x_n\}_{n \in N}$ is non-increasing monotonic and bounded below, and so there exists, $r \geq 0$ such that

$$\lim_{n\to\infty} d(x_n, x_{n+1}, a) = r = \lim_{n\to\infty} d(x_{n-1}, x_n, a) \quad (2.5)$$

Using the property of lower semi-continuous of ϕ, we have

$$\phi(r) \leq \lim_{n\to\infty} \inf \phi d(x_{n-1}, x_n, a).$$

Now, we claim that $r = 0$. In fact, taking upper limit as $n\to\infty$ on the following inequality and using (2.5) we have,

$$\Psi(d(x_n, x_{n+1}, a)) \leq \Psi(d(x_{n-1}, x_n, a)) - \phi(d(x_{n-1}, x_n, a)) \text{ implies}$$

$$\phi(r) \leq \Psi(r) - \phi(r).$$

i.e., $\phi(r) \leq 0$ implies $\phi(r) = 0$ and $\phi(r) = 0$ implies $r = 0$. Hence

$$\lim_{n\to\infty} d(x_n, x_{n+1}, a) = 0 \tag{2.6}$$

Next, we show that $\{x_n\}_{n\in N}$ is a Cauchy sequence. For this it is sufficient to prove that the subsequence $\{x_{2n}\}$ is a Cauchy sequence, but we suppose in contrary way that $\{x_{2n}\}$ is not a Cauchy sequence. Then, there is an $\varepsilon > 0$ for which can find two subsequences $\{x_{2mk}\}$ and $\{x_{2nk}\}$ and such that n_k is the smallest index for which $n_k > m_k > k$, $d(x_{2mk}, x_{2nk}, a) \geq \varepsilon$ and $d(x_{2mk}, x_{2nk-2}, a) < \varepsilon$.

Then (2.6) and the inequality

$$\begin{aligned}
\varepsilon \leq d(x_{2mk}, x_{2nk}, a) &\leq d(x_{2mk}, x_{2nk}, x_{2nk-2}) + d(x_{2mk}, x_{2nk-2}, a) + d(x_{2nk-2}, x_{2nk}, a) \\
&\leq d(x_{2mk}, x_{2nk}, x_{2nk-2}) + d(x_{2mk}, x_{2nk-2}, a) \\
&\quad + d(x_{2nk-2}, x_{2nk}, x_{2nk-1}) + d(x_{2nk-2}, x_{2nk-1}, a) \\
&\quad + d(x_{2nk-1}, x_{2nk}, a) \\
&= d(x_{2mk}, x_{2nk-2}, a) + d(x_{2nk-2}, x_{2nk-1}, a) + d(x_{2nk-1}, x_{2nk}, a)
\end{aligned}$$

Implies, $\lim_{n\to\infty} d(x_{2mk}, x_{2nk}, a) = \varepsilon$.

Also, (2.6) and the inequality

$$\begin{aligned}
d(x_{2mk}, x_{2nk}, a) &\leq d(x_{2mk}, x_{2nk}, x_{2mk+1}) + d(x_{2mk}, x_{2mk+1}, a) + d(x_{2mk+1}, x_{2nk}, a) \\
&= d(x_{2mk}, x_{2mk+1}, a) + d(x_{2mk+1}, x_{2nk}, a)
\end{aligned}$$

Gives $\qquad \varepsilon \leq \lim_{k\to\infty} d(x_{2mk+1}, x_{2nk}, a).$

So, (2.6) and the inequality

$$d(x_{2mk+1}, x_{2nk}, a) \leq d(x_{2mk+1}, x_{2mk}, a) + d(x_{2mk}, x_{2nk}, a) \text{ yields}$$

$$\varepsilon = \lim_{k\to\infty} d(x_{2nk}, x_{2mk+1}, a).$$

Taking $x = x_{2nk+1}, y = x_{2mk}$ in (2.1) and (2.4), we have

$$d(x_{2nk+2}, x_{2mk+1}, a) = \Psi(d(Tx_{2nk+1}, Sx_{2mk}, a))$$
$$\leq \Psi(M_1(Tx_{2nk+1}, Sx_{2mk}, a)) - \phi(M_1(Tx_{2nk+1}, Sx_{2mk}, a)),$$

where $M_1(Tx_{2nk+1}, Sx_{2mk}, a) = \max\{d(x_{2nk+2}, x_{2mk}, a), d(x_{2nk+1}, Tx_{2nk+1}, a), d(x_{2mk}, Sx_{2mk}, a), [(d(x_{2mk}, Tx_{2nk+1}, a) + d(x_{2nk+1}, Sx_{2mk}, a))]/2, [d(x_{2nk+1}, Tx_{2nk+1}, a) + d(x_{2mk}, Sx_{2mk}, a)/2, d(x_{2mk}, Sx_{2mk}, a)] [(1+d(x_{2nk+1}, Tx_{2nk+1}, a)) /(1+d(x_{2nk+1}, x_{2mk}, a))], d(x_{2nk+1}, Tx_{2nk+1}, a)[(1+d(x_{2mk}, Sx_{2mk}, a))/ (1+d(x_{2nk+1}, x_{2mk}, a))]\}$

for which $\lim_{k\to\infty} M_1(Tx_{2nk+1}, Sx_{2mk}, a) = \varepsilon$.

Hence, we have $\phi(\varepsilon) \leq \Psi(\varepsilon) - \phi(\varepsilon)$, which is a contradiction with $\varepsilon > 0$. It follows that $\{x_n\}_{n\in N}$ is a Cauchy sequence in X, and completeness of X ensures the convergence to a limit, say $z \in X$.

Now, we show that z is a common fixed point theorem of T and S. For this, using (2.1) we get,

$$\Psi(d(Tz, Sx_{2nk}, a)) \leq \Psi(M_1(Tz, Sx_{2nk}, a)) - \Psi(M_1(Tz, Sx_{2nk}, a))$$
$$= \Psi(\max\{d(z, x_{2nk}, a), d(z, Tz, a), d(x_{2nk}, Sx_{2nk}, a), [d(x_{2nk}, Tz , a) + d(z, Sx_{2nk}, a)]/2, [d(z, Tz, a) + d(x_{2nk}, Sx_{2nk}, a)]/2, d(x_{2nk}, Sx_{2nk}, a)[(1+d(z, Tz, a))/ (1+d(z, x_{2nk}, a))], d(z, Tz, a) [(1+d(x_{2nk}, Sx_{2nk}, a))/ (1+d(z, x_{2nk}, a))]\})$$
$$- \phi(\max\{d(z, x_{2nk}, a), d(z, Tz, a), d(x_{2nk}, Sx_{2nk}, a), [d(x_{2nk}, Tz, a) + d(z, Sx_{2nk}, a)]/2, [d(z, Tz, a) + d(x_{2nk}, Sx_{2nk}, a)]/2, d(x_{2nk}, Sx_{2nk}, a)[(1+d(z, Tz, a))/ (1+d(z, x_{2nk}, a))], d(z, Tz, a) [(1+d(x_{2nk}, Sx_{2nk}, a))/ (1+d(z, x_{2nk}, a))]\})$$

making $k \to\infty$, we have

$\Psi(d(z, Tz, a)) \leq \Psi(d(z, Tz, a)) - \phi(d(z, Tz, a))$, which yields $z = Tz$.

Further, we get

$$\Psi(d(Tz, Sz, a)) \leq \Psi(d(Tz, Sz, a)) - \phi(d(Tz, Sz, a))$$

$$= \Psi(\max\{d(z, z, a), d(z, z, a), d(z, Sz, a), [d(z, z, a) + d(z, Sz, a)]/2, [d(z, z, a) + d(z, Sz, a)]/2, d(z, Sz, a)[(1+d(z, z, a))/(1+d(z, z, a))], d(z, z, a)[(1+d(z, Sz, a))/(1+d(z, z, a))]\})$$

$$- \phi(\max\{d(z, z, a), d(z, z, a), d(z, Sz, a), [d(z, z, a) + d(z, Sz, a)]/2, [d(z, z, a) + d(z, Sz, a)]/2, d(z, Sz, a)[(1+d(z, z, a))/(1+d(z, z, a))], d(z, z, a)[(1+d(z, Sz, a))/(1+d(z, z, a))]\})$$

implies

$$\Psi(d(z, Sz, a)) \leq \Psi(d(z, Sz, a)) - \phi(d(z, Sz, a)),$$

which provides $z = Sz$. Hence z is a common fixed point of T and S.

For uniqueness, we suppose that y is another fixed point of T and S, and we have

$$\Psi(d(y, z, a)) \leq \Psi(d(Ty, Sz, a))$$

$$\leq \Psi(M_1(Ty, Sz, a)) - \phi(M_1(Ty, Sz, a))$$

$$= \Psi(d(y, z, a)) - \phi(d(y, z, a)) \text{ implies}$$

$$\phi(d(y, z, a)) = 0.$$

Therefore, $y = z$.

Remark 2.1: Theorem 2.1 is generalization of the result of Arya et al. [13].

Corollary 2.1: Now for $\Psi = I$ (identity) in theorem (2.1), we get the following corollary-

Let (X, d) be a complete 2-metric space and let $T, S : X \to X$ be two mappings. Assume that to every $x, y, a \in X$

$$d(Tx, Sy, a) \leq M_1(Tx, Sy, a) - \phi(M_1(Tx, Sy, a)),$$

where ϕ is defined as in theorem (1.1). Then there exists a unique point $z \in X$ such that $z = Tz = Sz$.

For $S = T$ we obtain the following corollary of theorem 2.1.

Corollary 2.2: Let (X, d) be a complete 2-metric space and let $T, S: X \to X$ be a mapping.

Assume that for every $x, y, a \in X$

$$\Psi(d(Tx, Ty, a)) \leq \Psi(M_1(Tx, Ty, a)) - \phi(M_1(Tx, Ty, a)),$$

where Ψ and ϕ are defined as in theorem (1.2). Then there exists a unique point $z \in X$ such that $z = Tz$.

Taking $M_1(Tx, Ty, a) = d(x, y, a)$ in theorem (2.1), we get the following generalization of the results as Dutta et al.[6].

Corollary 2.3: Let (X, d) be complete 2-metric space and let $T, S : X \to X$ be two mappings.

Assume that for every $x, y, a \in X$

$$\Psi(d(Tx, Sy, a)) \leq \Psi(d(x, y, a)) - \phi(d(x, y, a)),$$

where Ψ and ϕ are defined as in theorem (1.2). Then there exists a unique point $z \in X$ such that $z = Tz = Sz$.

Corollary 2.4: Let (X, d) be complete 2-metric space and let $T, S : X \to X$ be two mappings.

Assume that for every $x, y, a \in X$

$$\Psi(d(Tx, Sy, a)) \leq \Psi(N(Tx, Sy, a)) - \phi(N(Tx, Sy, a)),$$

where $N(Tx, Sy, a) = \max\{d(x, y, a), d(y, Sy, a)\,(\frac{1+d(x, Tx, a)}{1+d(x, y, a)})\}$

and Ψ and ϕ are defined as in theorem (1.2). Then there exists a unique point $z \in X$ such that $z = Tz = Sz$.

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