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Some Separation Axioms in Topological Spaces

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Abstract:

In this paper, we introduced the concepts of new separation axioms called SC^*-separation axioms and H^*-separation axioms by using SC^* and H^*- open sets in topological spaces. SC^*- separation axioms i.e. SC^*- C_0, SC^*- C_1, weakly SC^*- C_0 and weakly SC^*-C_1 and H^*-$T_{1/2}$, H^*-T_b, H^*- T_d-spaces. Also we obtained several properties of such spaces.

Keywords & Phrases: SC^*- C_0; SC^*- C_1; weakly SC^*- C_0; weakly SC^*- C_1; gH^*-closed; gH^*- open; rgH^*- open sets; H^*- $T_{1/2}$; H^*- T_b and H^*-T_d - spaces.

1. Introduction:

In this paper, we introduced the concepts of new separation axioms called SC^* and H^*-separation axioms i.e. SC^*- C_0, SC^*- C_1, weakly, SC^*- C_0, weakly SC^*- C_1 and H^*- $T_{1/2}$, H^*- T_b, H^*- T_d by using SC^* and H^*- open sets. Due to A. Chandrakala and K. Bala Deepa Arasi [1] and Nidhi Sharma et al. [2] in topological space and obtained several properties of such spaces. In 2024, K. Suthi Keerthana et al. [3] introduced a new separation axioms i.e. $\alpha b^* g\alpha$-closed sets, namely $cT_{\alpha b^* g\alpha}$- space, $T^*_{\alpha b^* g\alpha}$- space, $^*g\alpha T_{1/2}{}^{***}$- space, by using $\alpha b^* g\alpha$- open, $b^* g\alpha$- open sets and obtained several properties of such spaces.

2. Prerequisites:

Throughout this paper by a space X we mean it is a topological space (X, τ). If A is any subset of a space X, then cl(A), int(A) and $C(A)$ denote the closure, the interior and the complement of A respectively. The family of all semi-open (resp. pre-open, α-open, c^*- open) subsets of a space X is denoted by $SO(X)$ (resp. $PO(X)$, $\alpha O(X)$, $c^*O(X)$). Maheswari and Tapi [**4**] called a subset B of a space X as **feebly open** if there is an open set G such that $G \subseteq B \subseteq s\text{cl}(G)$. Later Janković and Reilly [**5**] observed that feebly open sets are precisely α-open sets.

Let us recall the following definitions:

Definition 2.1: A subset A of a topological space X is called

(1) **semi open** [6] if $A \subseteq \text{cl}(\text{int}(A))$.

(2) **α-open** [7] if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$.

(3) **pre-open** [8] if $A \subseteq \text{int}(\text{cl}(A))$.

(4) ***c**-open** [9] if $\text{int}(\text{cl}(A)) \subset A \subset \text{cl}(\text{int}(A))$.

(5) ***SC**-closed** [1] if $s\text{cl}(A) \subset U$, whenever $A \subset U$ and U is c^*-open in X.

The complement of semi-open (resp. α-open, pre-open, c^*-open) subset of a space X is called semi-closed (resp. **pre-closed, α-closed, *c**-closed**) set. $s\text{cl}(A)$ (resp. $p\text{cl}(A)$, $\alpha\text{cl}(A)$, $c^*\text{cl}(A)$) denote the **semi-closure** (resp. **pre-closure, α-closure, *c**-closure**) of the set A.

The complement of SC^*-closed set is called SC^*-open set.

Definition 2.2: A subset A of a topological space X is called

(1) **α^*-set** [10], if $\text{int}(\text{cl}(\text{int}(A))) = \text{int}(A)$.

(2) ***C*-set** [10], if $A = U \cap V$, where U is an open and V is an α^*-set in X.

(3) **αCg-closed** [11], if $\alpha\text{-cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is C-set in X.

(4) ***w*-closed** [12], if $\text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is semi-open in X.

(5) ***h*-closed** [13], if $s\text{-cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is w-open in X.

(6) ***hCg*-closed** [13], if $h\text{-cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is C-set in X.

(7) ***H**-closed** [2], if $h\text{-cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is hCg-open in X.

(8) ***H***g*-closed** [2], if $H^*\text{-cl}(A) \subset U$, whenever $A \subset U$ and U is open in X.

(9) ***gH**-closed** [2], if $H^*\text{-cl}(A) \subset U$, whenever $A \subset U$ and U is H^*-open in X.

The complement of αCg-closed (resp. w-closed, h-closed, hCg-closed, H^*-closed, H^*g-closed, gH^*-closed) set is said to be **αCg-open** (resp. ***w*-open, *h*-open, *hCg*-open, *H**-open**, ***H***g*-open, *gH**-open**) set. The intersection of all H^*-closed subsets of X containing A is called the **H^*-closure of A** and is denoted by **H^*-cl(A)**. The union of all H^*-open subsets of X in which are contained in A is called the **H^*-interior** of A and is denoted by **H^*-int(A)**. The family of H^*-open (resp. H^*-closed) sets of a topological space X is denoted by **$H^*O(X)$** (resp. **$H^*C(X)$**).

Definition 2.3: A topological space X is called

(1) $\boldsymbol{C_0}$ **(semi-** $\boldsymbol{C_0}$**)** if, for $x, y \in X, x \neq y$, there exists $G \in \tau$ ($SO(X)$) such that cl(G) ($s$cl(G)) contains only one of x and y but not the other;

(2) $\boldsymbol{C_1}$ **(semi-** $\boldsymbol{C_1}$**)** if, for $x, y \in X, x \neq y$, there exists $G, H \in \tau$ ($SO(X)$) such that $x \in$ cl(G) ($s$cl(G)), $y \in$ cl(H) ($s$cl(H)) but $x \notin$ cl(H) ($s$cl(H)) and $y \notin$ cl(G) ($s$cl(G));

(3) $\boldsymbol{w}$**-** $\boldsymbol{C_0}$ [**14**] if $\bigcap_{x \in X} \ker(x) = \phi$, where $\ker(x) = \cap \{G : x \in G \in \tau\}$;

(4) **weakly semi-** $\boldsymbol{C_0}$ if $\bigcap_{x \in X} s\ker(x) = \phi$, where $s\ker(x) = \cap \{G : x \in G \in SO(X)\}$;

(5) $\boldsymbol{R_0}$ [**15**] if $\text{cl}(\{x\}) \subseteq G$ whenever $x \in G \in \tau$;

(6) **semi-** $\boldsymbol{R_0}$ [**16**] if, for $x \in G \in SO(X)$, $s\text{cl}(\{x\}) \subseteq G$;

(7) **weakly** $\boldsymbol{R_0}$ [**14**] if $\bigcap_{x \in X} \text{cl}(\{x\}) = \phi$;

(8) **weakly semi-** $\boldsymbol{R_0}$ [**17**] if $\bigcap_{x \in X} s\text{cl}(\{x\}) = \phi$;

(9) **weakly pre-** $\boldsymbol{R_0}$ if $\bigcap_{x \in X} p\text{cl}(\{x\}) = \phi$;

(10) **weakly pre-** $\boldsymbol{C_0}$ if $\bigcap_{x \in X} p\ker(x) = \phi$, where $p\ker(x) = \cap \{G : x \in G \in PO(X)\}$;

(11) $\boldsymbol{\alpha}$**-space** [**18**] if every α-open set in it is open.

Maheswari and Prasad [**19**] introduced semi-T_i $(i = 0, 1, 2)$ axiom, which is weaker than T_i $(i = 0, 1, 2)$ axiom.

We use the following sets and classes for counter examples.

Let $X = \{a, b, c, d\}$, $Y = \{a, b, c\}$, $Z = \{a, b, c, d, e, f\}$

Let $\tau_1 = \{\phi, X, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$, $\sigma_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$,

$\eta_1 = \{\phi, Z, \{a, c, e\}, \{b, d, f\}\}$, $\sigma_2 = \{\phi, Y, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$,

$\tau_2 = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$,

$\sigma = \{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$.

Remark 2.4: (X, σ_1) is semi-C_0 but not C_0. (X, τ_2) is semi-C_1, C_0 but not C_1. (Y, σ_2) is semi-T_0 but not semi-C_0. (Z, η_1) is an α-space but not an α-C_0.

Theorem 2.5:

(1) Every C_1 (semi-C_1) space is a C_0 (semi-C_0).

(2) Every $C_0(C_1)$ space is a semi-C_0 (semi-C_1).

(3) Every R_0 space is a weakly R_0 [**14**].

(4) Every weakly R_0 space is weakly semi-R_0 [**17**].

Every semi-C_0(semi-C_1) space is semi-T_0(semi-T_1).

Proof: Omitted.

3. SC^*-C_0, SC^*-C_1, weakly SC^*-C_0 and weakly SC^*-R_0 spaces:

Now we introduce the following separation properties using SC^*-open sets in spaces.

Definition 3.1: A topological space X is called

(1) $\boldsymbol{SC^*}$**-**$\boldsymbol{C_0}$ if, for $x, y \in X$, $x \neq y$, there exists $G \in SC^*(X)$ such that $SC^*\text{cl}(G)$ contains only one of x and y but not the other;

(2) $\boldsymbol{SC^*}$**-**$\boldsymbol{C_1}$ if, for $x, y \in X$, $x \neq y$, there exists $G, H \in SC^*(X)$ such that $x \in SC^*$ cl(G), $y \in SC^*\text{cl}(H)$ but $x \notin SC^*\text{cl}(H)$ and $y \notin SC^*\text{cl}(G)$;

(3) **weakly** $\boldsymbol{SC^*}$**-**$\boldsymbol{C_0}$ if, $\bigcap_{x \in X} SC^*\ker(x) = \phi$, where $SC^*\ker(x) = \cap \{G : x \in G \in SC^*(X)\}$;

(4) **weakly** $\boldsymbol{SC^*}$**-**$\boldsymbol{R_0}$ if, $\bigcap_{x \in X} SC^*\text{cl}(\{x\}) = \phi$.

Theorem 3.2:

(1) Every SC^*-C_1 space is SC^*-C_0.

(2) Every SC^*-C_0 (SC^*-C_1) space is semi-C_0 (semi-C_1).

(3) Every weakly SC^*-R_0 space is weakly semi-R_0 and weakly pre-R_0.

(4) Every w-C_0 space is weakly SC^*-C_0.

(5) Every weakly SC^*-C_0 space is weakly semi-C_0 and weakly pre-C_0.

(6) Every SC^*-C_0 (SC^*-C_1) space is semi-T_0 (semi-T_1).

(7) Every weakly R_0 space is weakly SC^*-R_0.

(8) Weakly SC^*-R_0 ness and weakly SC^*-C_0 ness are independent notions.

Remark 3.3: (X, τ_2) is SC^*-C_0 but not SC^*-C_1. (X, σ_1) is semi-C_0, semi-C_1, but neither SC^*-C_1 nor SC^*-C_0. (Y, σ) is weakly semi-R_0 but not weakly SC^*-R_0. (X, τ_2) is weakly SC^*-C_0 but not weakly SC^*-R_0. (X, τ_1) is weakly SC^*-R_0 but not weakly SC^*-C_0.

Theorem 3.4: A topological space X is weakly SC^*-R_0 if and only if $SC^*\ker(x) \neq X$ for each $x \in X$.

Proof:

Necessity - If there is some $x_0 \in X$ with $SC^*\ker(x_0) = X$, then X is the only SC^*-open set containing x_0. This implies that $x_0 \in SC^*cl(\{x\})$ for every $x \in X$. Hence $\bigcap_{x \in X} SC^*cl(\{x\}) \neq \phi$, a contradiction.

Sufficiency - If X is not weakly SC^*-R_0, then choose some $x_0 \in \bigcap_{x \in X} SC^*\ker(x)$.

This implies that every α-neighborhood of x_0 contains every point of X. Hence $SC^*\ker(x_0) = X$.

Theorem 3.5: A topological space X is weakly SC^*-C_0 if and only if for each $x \in X$, there exists a proper SC^*-closed set containing x_0.

Proof:

Necessity - Suppose there is some $x_0 \in X$ such that X is the only SC^*-closed set containing x_0. Let U be any proper α-open subset of X containing a point x. This implies that $C(U) \neq X$. Since $C(U)$ is SC^*-closed, we have $x_0 \in C(U)$. So $x_0 \in U$. Thus $x_0 \in \bigcap_{x \in X} \ker(x)$ for any point x of X, a contradiction.

Sufficiency - If X is not weakly SC^* - C_0, then choose $x_0 \in \bigcap_{x \in X} SC^*\text{ker}(x)$. So x_0 belongs to $SC^*\text{ker}(x)$ for any $x \in X$. This implies that X is the only SC^*- open set, which contains the point x_0, a contradiction.

Theorem 3.6: Every SC^*- C_0 (SC^*- C_1) space is weakly SC^*- C_0.

Proof: If $x, y \in X$ such that $x \neq y$, where X is an SC^*- C_0 space, then without loss of generality, we can assume that there exists $U \in SC^*(X)$ such that $x \in SC^*\text{cl}(U)$ but $y \notin SC^*\text{cl}(U)$. This implies that $U \neq \phi$. Hence, we can choose z in U. Now $SC^*\text{ker}(z) \cap SC^*\text{ker}(y) \subseteq U \cap (SC^*\text{cl}(U)) \subseteq (SC^*\text{cl}(U)) \cap C(SC^*\text{cl}(U)) = \phi$.

Hence $\bigcap_{x \in X} \text{ker}(x) = \phi$. Since every SC^*- C_1 space is also SC^*- C_0, it is also clear that every SC^*- C_1 space is weakly SC^*- C_0.

Remark 3.7: The converse of the above theorem need not be true since (Z, η_1) is weakly SC^*- C_0 but not SC^*- C_0. The space (Y, σ) is both SC^*- C_0 and weakly SC^*- C_0 but not SC^*- C_1.

Theorem 3.8: The property of being an SC^*- C_0 space is not hereditary.

Proof: Consider the space (Y, σ_1). Let $S = \{a, c\}$ and σ_1^* be the relative topology on S. It is easy to verify that (Y, σ_1) is SC^*- C_0 but it's subspace (S, σ_1^*) is not SC^*- C_0.

4. H^*- Separation Axioms:

Remark 4.1: Every α- closed (resp. α- open) set is H^*- closed (resp. H^*- open) set.

For definitions stated above, we have the following diagram:

$$\begin{array}{ccccccc} \text{closed} & \rightarrow & \alpha\text{-closed} & \rightarrow & g\alpha\text{-closed} & \rightarrow & \alpha g\text{-closed} \\ & & \downarrow & & \downarrow & & \downarrow \\ & & H^*\text{-closed} & \rightarrow & gH^*\text{-closed} & \rightarrow & H^*g\text{-closed.} \end{array}$$

However the converses of the above are not true as may be seen by the following examples:

Example 4.1.1: Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$. Then $A = \{c\}$ is α-closed set as well as H^*-closed set but not closed set in X.

Example 4.1.2: Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$. Then the set $A = \{c\}$ is gH^*-closed set but not closed set in X.

Remark 4.2:

(i) A subset A of X is H^*g-open in X iff $F \subset H^*\text{-cl}(A)$ whenever $F \subset U$ and U is closed in X.

(ii) A subset A of X is gH^*-open in X iff A is g-closed (resp. g-open) in X.

5. H^*-T_1, H^*-$T_{1/2}$, H^*-T_b, H^*-T_d-Spaces:

Definition 5.1: A topological space X is said to be

(1) T_1 (resp. H^*-T_1), if for any distinct pair of points x and y in X, there exists an open (resp. H^*-open) set U in X containing x but not y and open (resp. H^*-open) set V in X containing y but not x.

(2) A $T_{1/2}$ [6], if every g-closed set is closed.

(3) A H^*-$T_{1/2}$, if every gH^*-closed set is H^*-closed.

(4) A H^*-T_b, if every H^*g-closed set in X is closed.

(5) A H^*-T_d, if every H^*g-closed set in X is g-closed.

$$\begin{array}{ccccccc} T_1 & \rightarrow & T_{1/2} & \leftarrow & H^*\text{-}T_b & \rightarrow & H^*\text{-}T_d \\ \downarrow & & \downarrow & & & & \\ H^*\text{-}T_1 & \rightarrow & H^*\text{-}T_{1/2} & & & & \end{array}$$

It can be explained by the following example:

Example 5.1.1: An H^*-T_b space need not be H^*-T_1. Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$. Since $\{a, b\}$ is not H^*-closed it is not H^*-T_1 and hence it is not T_1. However the family of all H^*g-closed sets coincides with one of all closed sets and hence it is H^*-T_b.

Theorem 5.2:

(i) X is a $\boldsymbol{T_{1/2}}$ iff for each $x \in X$, $\{x\}$ is open or closed in X.

(ii) X is a $\boldsymbol{H^*\text{-}T_{1/2}}$ space iff for each $x \in X$, $\{x\}$ is H^*-open or H^*-closed in X, i.e., X is a $\boldsymbol{H^*\text{-}T_{1/2}}$ iff a space (X, τ^{H^*}) is $\boldsymbol{T_{1/2}}$-space.

Theorem 5.3:

(i) If A is H^*g-closed, then $H^*\text{-cl}(A) - A$ does not contain non-empty closed set.

(ii) For each $x \in X$, $\{x\}$ is closed or its complement $X - \{x\}$ is H^*g-closed in X.

(iii) For each $x \in X$, $\{x\}$ is H^*-closed or its complement $X - \{x\}$ is gH^*-closed in X.

Theorem 5.4:

(i) Every $H^*\text{-}T_b$ space is $H^*\text{-}T_d$ and $T_{1/2}$.

(ii) Every T_i-space is $H^*\text{-}T_i$, where $i = 1, 1/2$.

(iii) Every $H^*\text{-}T_i$ space is $H^*\text{-}T_{1/2}$.

Proof:

(i) It is obtained from **Definition 5.1**. [(ii), (iv) and (v)] and **Remark 4.2, (i), (ii)**.

(ii) Let X be a T_1 (resp. $T_{1/2}$)-space and let $x \in X$. Then $\{x\}$ is closed (resp. open or closed by **Theorem 5.2 (i)**. Since every open set is H^*-open, $\{x\}$ is H^*-closed (resp. H^*-closed or H^*-open) in X. This implies that X is T_1 (resp. $T_{1/2}$ by **Theorem 5.2 (ii)**. Therefore X is $H^*\text{-}T_1$ (resp. $H^*\text{-}T_{1/2}$).

(iii) Let X be a $H^*\text{-}T_1$ space. Then X is T_1. By **Theorem 5.3** of **Levine** [20] X is $T_{1/2}$ and hence X is $H^*\text{-}T_{1/2}$.

Proposition 5.5:

(i) If X is $\boldsymbol{H^*}$- $\boldsymbol{T_b}$ then for each $x \in X$, $\{x\}$ is H^*-closed or open in X.

(ii) If X is $\boldsymbol{H^*}$- $\boldsymbol{T_d}$ then for each $x \in X$, $\{x\}$ is H^*-closed or g-open in X.

Proof:

(i) Suppose that, for an $x \in X$, $\{x\}$ is not H^*-closed. By **Theorem 5.2 (iii)** and **Remark 4.2 (i)** and **(ii)**, $X - \{x\}$ is H^*g-closed set. Therefore $X - \{x\}$ closed by using assumption and hence $\{x\}$ is open.

(ii) Suppose that, for a $x \in X$, $\{x\}$ is not closed. By **Theorem 5.3 (ii)**, $X - \{x\}$ is H^*g-closed set. Therefore by using assumption $X - \{x\}$ is g-closed and hence $\{x\}$ is g-open.

6. Separation Axioms H^*- T_b and H^*- T_d of Spaces are Preserved under Homomorphisms:

Definition 6.1: A map $f: X \to Y$ is said to be

(1) **pre $\boldsymbol{H^*}$-closed,** if for each H^*-closed set of X, $f(F)$ is H^*-closed set in Y.

(2) **$\boldsymbol{H^*}$-irresolute,** if for each H^*-closed set F of Y, $f^{-1}(F)$ is H^*-closed in X.

Theorem 6.2:

(i) A map $f: X \to Y$ is pre H^*-closed (resp. pre H^*-open) iff its induced map $f: (X, \tau^{H^*}) \to (Y, \sigma^{H^*})$ is a closed (resp. open) map.

(ii) A map $f: X \to Y$ is H^*-irresolute pre H^*-closed then for every gH^*-closed set A of Y, $f^{-1}(A)$ is H^*g-closed.

Theorem 6.3:

(i) A map $f: X \to Y$ is a homomorphism, then f is a H^*-homomorphism.

(ii) If X is H^*- T_b (resp. H^*- T_d) and $f: X \to Y$ is a homomorphism, then Y is H^*- T_b (resp. H^*- T_d).

Proof:

(i) Since $f : X \rightarrow Y$ is a homomorphism, then f and f^{-1} are both open and H^*-continuous bijection. It follows from **Theorem 4.16** of **Noiri [21]** that f and f^{-1} are H^*-irresolute. Therefore, f is H^*-irresolute and f is H^*-closed.

(ii) Let $f : X \rightarrow Y$ is a homomorphism and let F be an H^*g-closed set of Y. Then by (i) $f^{-1} : Y \rightarrow X$ is a continuous and pre H^*-closed bijection. Hence by **Theorem 6.2 (i)**, $f^{-1}(F)$ is H^*g-closed in X. Since X is H^*- T_b (resp. H^*- T_d), $f^{-1}(F)$ is closed (resp. g-closed) in X. Since f is closed onto (resp. closed and continuous) map, F is closed (resp. g-closed) by **Theorem 6.1** of **Levine [20]** in Y. Hence Y is H^*- T_b (resp. H^*- T_d).

Conclusion:

The research in topology over last two decades has reached a high level in many directions. By researching generalizations of closed sets, some new separation axioms have been founded and they turn out to be useful in the study of digital topology. Therefore, ***SC**** and ***H****-separation axioms are defined by using ***SC**** and ***H****-closed sets will have many possibilities of applications in **digital topology and computer graphics**.

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