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A Outcomes Study of Vibrations of Moments of Damped Circular Plate

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Abstract:

This representative model will be helpful in designing plate materials and devices where structures are use under temperature due to which features of material undergo change in vibrating modes. This works describes the vibrations of circular plates under damping and temperature factors. Method has employed to observe the moments of vibrations.

Keywords: Damping, plates , transverse, moments, elasticity.

Introduction:

Vibrations of thin plates under temperature have importance in aircraft, machine designs, astronautically engineering. In these problems the damped frequency on plates of circular shapes are of great importance. Under the distribution of the temperature the structure develops thermal stresses, which can be large

enough to cause yielding of the material. The damping effect will be helpful in controlling the frequency parameter and the amplitude of vibrations. Investigators [1- 3] have considered the free vibrations of isotropic non-homogeneous / homogeneous plate of variable thickness resting on elastic foundation. The non-dimensional frequency parameter and temperature established by Dhotarad and Ganesan [4]. Rao and others [5] studied the effect of a constant thermal gradient on the frequencies of tapered rectangular plate with different boundary conditions by Galerkin's method. Kumar [6] analyzing the influence of thermally induced non-homogeneity on the frequencies of a linearly tapered infinite plate resting on elastic foundation by Frobenius method. Shua and others [7] applied the 2-dimensional least-square-based finite difference (LSFD) method for solving free vibration problems with simply supported and clamped edges. Nesterov [8] observed square homogeneous plate clamped along its contour by resonance method. Sharma et al. [9] have studied the vibration of visco-elastic isotropic square plate with thermal effect on two direction varying thickness by using Rayleigh-Ritz technique with a two term deflection function. Investigators [10-11] have analyzing the 2D thermal effect on vibration of non-homogeneous square plate of exponentially (in x direction) varying thickness having clamped boundary by Rayleigh Ritz technique. Sharma & others [12] have observed the effect of thermal gradient on vibration of non-homogenous square plate with varying thickness in x or y directions by Rayleigh-Ritz technique. Meera & others [15] have note the large amplitude free vibration behavior of uniform moderately thick square plates with axially immovable edges.

Analysis:

The governing equation of motion in non-dimensional variables under concerned polar coordinates [17, 14] is given by

$$[\bar{E}]\frac{\partial^4 W}{\partial R^4}+\left[2\frac{\partial \bar{E}}{\partial R}+\frac{\bar{E}}{R}\right]\frac{\partial^3 W}{\partial R^3}+\left[\frac{\partial^2 \bar{E}}{\partial R^2}+\left(\frac{2+\nu}{R}\right)\frac{\partial \bar{E}}{\partial R}-\frac{\bar{E}}{R^2}\right]\frac{\partial^2 W}{\partial R^2}$$
$$+\left[\frac{\partial^2 \bar{E}}{\partial R^2}\left(\frac{\nu}{R}\right)+\left(\frac{1}{R^2}\right)\frac{\partial \bar{E}}{\partial R}-\frac{\bar{E}}{R^3}\right]\frac{\partial^2 W}{\partial R^2}$$
$$+\left[\left\{\frac{12(1-\nu^2)\bar{\rho}a^2}{H^2}\right\}\frac{\partial^2 W}{\partial t^2}+\left\{\frac{12(1-\nu^2)K}{H^3}\right\}\frac{\partial W}{\partial t}\right]=0 \qquad (1)$$

If, taking as the reference temperature, the temperature at the end of the plate i.e. at $R = 1$, the modulus variation [4 - 6, 14] as :

$$\bar{E}(X) = \bar{E}_0\{1 - \zeta(1 - R)\} \tag{2}$$

Where, $\zeta = \Phi t_0$, $(0 \leq \zeta < 1)$ is called the thermal gradient, let $\zeta = \alpha$.

Let the harmonic damped vibration is considered as

$$W(X, t) = \overline{W}(X, t)e^{-\eta t}\,\mathrm{Cos}pt \tag{3}$$

Solved the equation (1, 2, 3) we obtained as with $\eta = \gamma$

$$[\{1 - \alpha + \alpha R\}]\frac{\partial^4 \overline{W}}{\partial R^4} + \left[2\alpha + \frac{2\{1 - \alpha + \alpha R\}}{R}\right]\frac{\partial^3 \overline{W}}{\partial R^3}$$

$$+ \left[\frac{(2+\nu)\alpha}{R} - \frac{\{1 - \alpha + \alpha R\}}{R^2}\right]\frac{\partial^2 \overline{W}}{\partial R^2} + \left[-\frac{\alpha}{R^2} + \frac{\{1 - \alpha + \alpha R\}}{R^2}\right]\frac{\partial \overline{W}}{\partial R}$$

$$- \{1 - \alpha + \alpha R\}[(D_p^2 I^{*2}) + (F^2 I^*) - n_F I^*]\,\overline{W}(R) = 0 \tag{4}$$

Solution and its Convergence:

Let the solution for $\overline{W}$ is assured in the series from as:

$$\overline{W}(R) = \sum_{\lambda=0}^{\infty} a_\lambda R_*^{C+\lambda} \quad \text{with } a_0 \neq 0 \tag{5}$$

Where C is the exponent of singularity.

Compute equation (4) & (5) we gets as

$$\sum_{\lambda=0}^{\infty} a_\lambda \big\langle [L_1(\lambda)]R^{C+\lambda-4} + [L_2(\lambda)]R^{C+\lambda-3} + [L_3(\lambda)]R^{C+\lambda-2} + [L_4(\lambda)]R^{C+\lambda-1}$$

$$+ [L_5(\lambda)]R^{C+1} + [L_6(\lambda)]R^{C+1+1} + [L_7(\lambda)]R^{C+1+2} + [L_8(\lambda)]R^{C+1+3}\big\rangle = 0 \tag{6}$$

In equation (6) we equating coefficient of the lowest power of R, the indicial roots are observed. For next power of R, the constants a_2, are indeterminate for $C = 0$ and $a_{3\text{-}12}$ taken as along with a_0 and a_2. The remaining unknown constant $a_\lambda(\lambda = 13, 14 .....)$ are determined from the recurrence relation.

Let the following solution at $c=0$ is

$$W = a_0\left[1+\sum_{\lambda=4}^{\infty} A_\lambda R^{C+\lambda}\right] + a_2\left[R^2+\sum_{\lambda=4}^{\infty} B_\lambda R^{C+\lambda}\right] \tag{7}$$

It is evident that no new solution will arise corresponding to other value of c. The solution corresponding to these values of c are already contained in the solution (7). The technique used by Lamb [16] has been applied for convergence of solution (7). If $\lambda \to \infty$ one finds that equation (7) is uniformly convergent $(0 \leq R \leq 1$ when $|\mu| < 1)$. We compute equation (7) & (6) then gets recurrence relation.

Conditions and Equation:

The frequency equations for clamped and -simply supported of circular plates are obtained by using the following boundary conditions for [A] C-Plates and [B] SS-Plates

[A] Let for this plate clamped [17] at both the edges $R=0$ and $R=a$, the deflection and slope of the plate element at the edges should be zero.

$$\overline{W}\Big|_{R=0,1} = \overline{W}^{I}\Big|_{R=0,1} = 0 \tag{8}$$

By using equation (7) and (8) the frequency equation for a C-plate is obtained as

$$\begin{vmatrix} J_1(\Omega) & J_2(\Omega) \\ J_3(\Omega) & J_4(\Omega) \end{vmatrix} = 0 \tag{9}$$

$$\left.\begin{aligned} \text{Where, } J_1(\Omega) &= 1+\sum_{\lambda=4}^{\infty} A_\lambda \ , \qquad J_2(\Omega) = 1+\sum_{\lambda=4}^{\infty} B_\lambda \\ J_3(\Omega) &= 1+\sum_{\lambda=4}^{\infty} \lambda\, A_\lambda \ \text{ and } J_4(1,\Omega) = 2+\sum_{\lambda=4}^{\infty} \lambda\, B_\lambda \end{aligned}\right\}$$

[B] For an circular plate clamped at simply supported [17] at $R=1$, the boundary conditions are

$$\overline{W}\Big|_{R=0,1} = \overline{W}^{I}\Big|_{R=0} = 0 = \overline{W}^{II}\Big|_{R=1} = 0 \tag{10}$$

Applying equation (10) on (7), the equation for (SS) - plate as

$$\begin{vmatrix} J_1(\Omega) & J_2(\Omega) \\ J_5(\Omega) & J_6(\Omega) \end{vmatrix} \tag{11}$$

Where, $J_5(\Omega) = \sum_{\lambda=4}^{\infty} A_\lambda \, \lambda(\lambda - 1)$ and $J_6^I(\Omega) = \sum_{\lambda=4}^{\infty} \lambda(\lambda - 1) B_\lambda$

Using equation (7) with B.C. at $W = 0$ & $R = 1$ under $a_0 = 1$, we gets Moments function and obtained result.

Where, $\overline{M}\Big|_{R=0,1} = -D\left[\frac{\partial^2 W}{\partial R^2} + (\nu/r)\frac{\partial W}{\partial R}\right]\Big|$ and $\overline{M} = \frac{M_R}{M_0}$

Results and Discussion:

Results for an non-homogeneous, circular elastic plate of uniform thickness have been computed from the equations (9) and (11) when the temperature factor varies as linearly, using computer programming for combinations of α, nf & Dp. Under let, the Poisson's ratio ν (0.3) and thickness (0.1) of the plate has been assumed to remain constant. The variation of relative parameters for 4- modes of vibration have been computed for C and SS, plates for values of α & Dp shown in figures from one to four. If the thermal gradient and damping parameters are taken as zero for reducing the problem to well known isotropic homogeneous plate of uniform thickness, the results so obtained are in good agreement with the results obtained by Jain and Soni [13, 17]. The observations as the obtained parameter variations with the effect of R for heated as well as unheated plates shown in figures.

Result Figures:

Graph:

Figure 1 to 4 show : W with X for 4 Mode of Vibrations at Dp

[α = 0.1, Dp = 0.0012, 0.03, 0.3, 1.2 for C & SS-Plates]

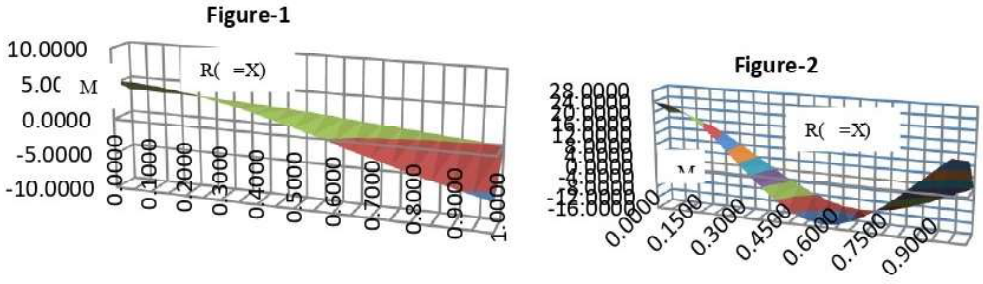


Figure-3

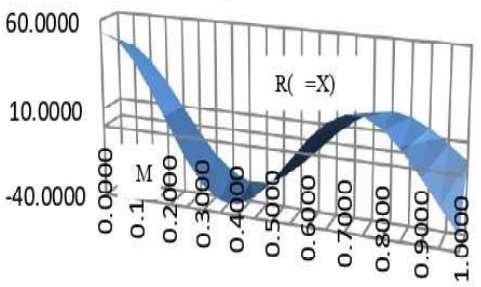
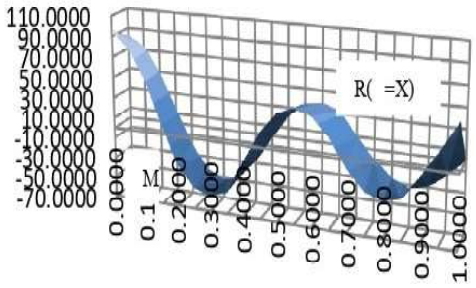


Figure-4



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