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Some remarks on Topological Characterisations of a *p*-irresolute homeomorphism

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Abstract:

In this paper we presented different types of continuous functions defined in terms of different types of newly defined open sets and we also present properties of irresolute functions from one topological space into another. We have present the most important characteristics properties of continuous functions, homeomorphism, pre-continuous functions, semi-continuous functions. We have present most important result of a p-irresolute homeomorphism.

Keywords: Continuous function, pre-continuous function, semi-continuous function, α-continuous function, β-continuous function, homeomorphism and *p*-irresolute homeomorphism.

§1. Preliminaries:

First of all we discuss fundamental topological concepts in this paper. We have also discuss definitions, properties and different types of continuous functions defined in terms of newly defined open sets and important results for easy reference.

Definition (1.1): Let (X, T) and (Y, T^1) be topological spaces and let $f : X \to Y$. Then f is said to be continuous at $x_0 \in X$, iff to every T^1 - n'hood M of $f(x_0)$ there exists a T - n'hood N of x_0 such that $f(N) \subset M$.

f is said to be continuous on X if it is continuous at every point of X.

Some characteristic properties of continuous functions are as follows :

Theorem (1.1): Let $f : (X, T) \to (Y, T^1)$ be a function. Then the following statements are equivalent :

(i) f is continuous on X.

(ii) $f^{-1}(V)$ is open in X for every open set V in Y.

(iii) $f^{-1}(F)$ is closed in X for every closed set F in Y.

(iv) $f(T\text{ - cl } A) \subseteq T^1\text{ - cl } f(A)$ for every subset A of X.

Definition (1.2): Let $f : (X, T) \to (Y, T^1)$. Then

(a) f is said to be an open function if $f(U)$ is open in Y for every open set U in X.

(b) f is called a homeomorphism if f is one-one function of X onto Y which is both open and continuous.

A characteristic property of a homeomorphism is as follows:

Theorem (1.2): Let $f : (X, T) \to (Y, T^1)$ be a function. Then a one-one onto function f is a homeomorphism iff $f(T\text{ - cl } A) = T^1\text{ - cl } f(A)$ for every subset A of X.

Definition (1.3): Let $f : X \to Y$ be a function. Then f is said to be pre-continuous at $x \in X$ if for every open set V containing $f(x)$ there exists a pre-open set L containing x such that $f(L) \subset V$.

If f is pre-continuous at every point $x \in X$ then f is said to be pre-continuous on x.

Characteristic properties of pre-continuous functions are as follows:

Theorem (1.3): Let $f : (X, T) \to (Y, T^1)$ be a function. Then the following statements are equivalent:

(i) f is pre-continuous on X.

(ii) $f^{-1}(V)$ is pre-open in X for every open set V in Y.

(iii) $f^{-1}(F)$ is pre-closed in X for every closed set F in Y.

(iv) $f(p\text{ - cl } A) \subseteq T^1\text{ - cl } f(A)$ for every subset A of X.

Remark (1.1): Every continuous function is pre-continuous but the converse may not be true.

Now, we see that s-continuous, α-continuous and β-continuous functions may be defined in similar manner.

Definition (1.4): Let $f : X \to Y$ be a function from a topological space (X, T) into another topological space (Y, T) and let $x \in X$. Then f is said to be semi-continuous (resp., α-continuous, β-continuous) at x if for every open set V containing $f(x)$ in Y there exists a semi-open (resp., α-open, β-open) subset L of X containing x such that $f(L) \subseteq V$. f is semi-continuous (resp., α-continuous, β-continuous) on X if it is semi-continuous (resp., α-continuous, β-continuous) at every point of X.

Characteristic properties of semi-continuous functions are as follows :

Theorem (1.4): Let $f : (X, T) \to (Y, T^1)$ be a function. Then the following statements are equivalent :

(i) f is semi-continuous (resp., α-continuous, β-continuous) on X.

(ii) $f^{-1}(V)$ is semi-open (resp., α-open, β-open) in X for every open set V in Y.

(iii) $f^{-1}(F)$ is semi-closed (resp., α-closed, β-closed) in X for every closed set F in Y.

(iv) $f^{-1}(s\text{ - cl}A)$ (resp., $\alpha\text{ - cl}A$, $\beta\text{ - cl}A$) $\subseteq T^1\text{ - cl } f(A)$ for every subset A of X.

We also have the following :

Remark (1.2): f is continuous on $X \Rightarrow f$ is p-continuous, S-continuous, α-continuous and β-continuous.

The converse of the result may not be true.

§ 2. Topological Characterisations of a *p*-irresolute homeomorphism:

In this section, we discuss some generalized topological concepts such as p-irresolute homeomorphism and their characteristics properties.

Definition (2.1): Let $f : (X, T) \to (Y, T^1)$ be a function. Then f is said to be p-irresolute (resp., s-irresolute, α-irresolute, β-irresolute) on X iff $f^{-1}(V)$ is p-open (resp., s-open, α-open, β-open) in X for every p-open (resp., s-open, α-open, β-open) set V in Y.

Now we have the following characteristic property:

Theorem (2.1): Let $f : (X, T) \to (Y, T)$ be a function. Then the following statements are equivalent:

(i) f is p-irresolute on x.

(ii) $f^{-1}(F)$ is p-closed in X for every p-closed set F in Y.

(iii) $f(p\text{ - cl }A) \subseteq p\text{ - cl }F(A)$ for every subset A of X.

Note (2.1): Similar results can be proved for s-irresolute (resp., α-irresolute and β-irresolute) functions.

Irresolute homeomorphism can be defined as follows:

Definition (2.2): Let $f : (X, T) \to (Y, T^1)$ be a mapping.

Then f is said to be p-irresolute homeomorphism if

(i) f is one-one

(ii) f is onto

(iii) both f and f^{-1} are p-irresolute mappings.

Note (2.2): Similar definition can be defined for s-irresolute, α-irresolute and β-irresolute homeomorphism.

Theorem (2.2): A mapping $f : X \to Y$ is p-irresolute homeomorphism iff

$$f(p\text{ - cl } A) = p\text{ - cl } f(A)$$

for every subset A of X.

Proof: f is p-irresolute homeomorphism iff f is one-one, onto and both f and f^{-1} are p-irresolute.

Now f is p-irresolute iff

$$f(p\text{ - cl } A) \subseteq p\text{ - cl } f(A) \tag{2.2.1}$$

Again f^{-1} is p-irresolute iff

$$f^{-1}(p\text{ - cl } B) \subseteq p\text{ - cl } f^{-1}(B) \tag{2.2.2}$$

for every subset B of Y.

Now we take $B = f(A)$ for $A \subseteq X$ use the fact that f is one-one, onto.

So we have

$$f^{-1}(p\text{ - cl} f(A)) \subseteq p\text{ - cl } f^{-1}(f(A))$$

$$\Rightarrow \quad f(f^{-1}(p\text{ - cl } f(A))) \subseteq f(p\text{ - cl } A)$$

$$\Rightarrow \quad p\text{ - cl} f(A) \subseteq f(p\text{ - cl } A) \tag{2.2.3}$$

Thus combining (2.2.1) and (2.2.3) we have

f is p-irresolute homeomorphism iff

$$f(p\text{ - cl } A) = p\text{ - cl} f(A)$$

for every subset A of X.

Note (2.3): Similar result can be proved for s-irresolute mapping.

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