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Specific Points and Sets for Some Generalized Concepts

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Abstract:

In this paper, first of all we present preliminary, definitions and results.

We also present some specific points and set of a topological space and other generalized concepts such as p -open and s -open sets. We investigate the most important characteristics property of a p -regular, s -regular and super p -regular. We have a generalized version of main results.

Keywords: s -open, s -closed, p -open, p -closed, p -regular-open, s -regular-open, p -irresolute, p -frechet, p -super-frechet, a quasi- p -frechet, p -housdorff, p -regular, super- p -regular.

§1. Preliminaries: First of all, we mention preliminary, definitions and results for easy reference.

Definition (1.1): A subset A of a topological space (X, T) is said to be

- (i) a semi open set (or s -open) if $A \subseteq \text{cl}(\text{int } A)$.
- (ii) a semi closed set (or s -closed) if $\text{int}(\text{cl } A) \subseteq A$
(or equivalently A^C is s -open).
- (iii) a pre-open set (or p -open) if $A \subseteq \text{int}(\text{cl } A)$.
- (iv) a pre-closed set (or p -closed) if $\text{cl}(\text{int } A) \subseteq A$
(or equivalently A^C is pre-open).
- (v) pre-regular open if $A = \text{int}(\text{cl } A)$ and
- (vi) semi-regular open if $A = \text{cl } \text{int } A$.

Now we mention some characteristics properties of these concepts:

Theorem (1.1): A non-empty subset A of a topological space (X, T) is

- (i) p -open iff for every closed set F containing A there exists an open set U such that $A \subseteq U \subseteq F$.
- (ii) A subset $A \subseteq X$ is s -open iff there exists an open set U of X such that $U \subseteq A \subseteq \text{cl}(U)$.

Corollary (1.1): Every open set is p -open and s -open but the converse is not true, in general.

Corollary (1.2): Arbitrary union of p -open (resp., s -open) sets is p -open (resp., s -open).

Corollary (1.3): Arbitrary intersection of p -closed (resp., s -closed) subsets is p -closed (resp., s -closed).

Definition (1.2): Let $f: (X, T) \rightarrow (Y, T^1)$ be a function. Then f is said to be p -irresolute (resp., s -irresolute) on X iff $f^{-1}(V)$ is p -open (resp., s -open) in X for every p -open (resp., s -open) set V in Y .

Definition (1.3): Let $f : (X, T) \rightarrow (Y, T^1)$ be a mapping. Then f is said to be p -irresolute (resp. s -irresolute) homeomorphism if

- (i) f is one-one
- (ii) f is onto
- (iii) both f and f^{-1} are p -irresolute (resp. s -irresolute) mappings.

§2. Some Specific Points: Based on separation axioms here we define some specific points and sets for a topological space and other generalized concepts.

Definition (2.1): A point p of a topological space (X, T) is called a Frechet (resp., p -Frechet, s -Frechet) point if for every point $q \neq p$ there exists an open (resp., p -open, s -open) subset G of X such that $q \in G, p \notin G$.

Definition (2.2): A point p of a topological space (X, T) is called a super Frechet (resp., p -super-Frechet, s -super-Frechet) point if for every point $q \neq p$ there exists open (resp., p -open, s -open) sets G and H such that

$$q \in G, p \notin G, p \in H, q \notin H.$$

In general, the above two concepts are different.

We prove the following:

Theorem (2.1): If p is Frechet (resp., p -Frechet, s -Frechet) point of a topological space (X, T) then $\{p\}$ is closed (resp., p -closed, s -closed) in X .

Proof: Let p be a Frechet (resp., p -Frechet, s -Frechet) point in (X, T) . Then for each $q \neq p$ there exists an open (resp., p -open, s -open) set G_q such that $q \in G_q, p \notin G_q$. Let $G = \cup G_q$. Then G is open (resp., p -open, s -open) in X . We claim that $G = \{p\}^C$. For, $q \in \{p\}^C \Rightarrow$ there exists a point $t \neq p$ such that $q \in G_t$ and so $q \in G$. Again $q \in G \Rightarrow q \in G_t$ for some $t \neq p$ and G_t is an open (resp., p -open, s -open) set not containing p . So, $q \neq p$ which means that $q \in \{p\}^C$. Hence $\{p\}^C = G$ and this proves that $\{p\}$ is closed.

Note (2.1): From the above result it follows that if every point of X is a Frechet (resp., p -Frechet, s -Frechet) point then X is a T_1 -space (resp., p - T_1 -space, s - T_1 -space). In this case the above two concepts coincide.

Definition (2.3): A point p of topological space (X, T) is called a quasi- p -Frechet (resp., s -Frechet) point if for every point $q \neq p$ there exists an open set G and a p -open set H such that $p \in G, q \notin G, q \in H, p \notin H$.

Definition (2.4): A point p of topological space (X, T) is called Hausdorff (resp., p -Hausdorff, s -Hausdorff) point if for every point $q \neq p$ there exist open (resp., p -open, s -open) sets G and H such that $p \in G, q \notin G, G \cap H = \phi$.

Definition (2.5): A point p of a topological space (X, T) is called a super regular (resp., p -regular, s -regular) point iff for every open (resp., p -open, s -open) set F not containing p there exists open sets G and H such that

$$F \subset G \text{ and } p \notin G.$$

Definition (2.6): A point p of a topological space (X, T) is called a super regular (resp., super p -regular, super- s -regular) point iff for every closed (resp., p -closed, s -closed) set F not containing p there exist open (resp., p -open, s -open) sets G and H such that

$$F \subset G, p \in H \text{ and } G \cap H = \phi.$$

Characteristic properties of the above concepts areas follows:

Theorem (2.2): A point p of a topological space (X, T) is a Regular (resp., p -regular, s -regular) point iff for every open (resp., p -open, s -open) set U containing x there exists a closed (resp., p -closed, s -closed) set F such that $x \in F \subset U$.

Proof: We prove the result for a p -regular point.

Suppose that x is a regular point of X and let U be an open set containing x . Then U^C is a closed set not containing x . So there exists an open set W such that $U^C \subset W$ and $x \notin W$. If we put $W^C = F$. Then F is closed and $x \in F \subseteq U$.

Conversely, suppose that the given condition is satisfied for some $x \in X$. Let L be a closed set not containing x . Then $x \in L^C$ and L^C is p -open. So there exists a p -closed set F such that $x \in F \subseteq L^C$. This gives $L \subseteq F^C$ and $x \notin F^C$. So, x is a p -regular point of X .

Corollary (2.1): If X is a T_1 -space (resp., p - T_1 -space). Then every point of X is a regular (resp., p -regular) point.

Similarly, we can prove the following result:

Theorem (2.3): A point p of a topological space (X, T) is a super regular (resp., super p -regular) point iff for every open (resp., p -open) set U containing x there exists an open (resp., p -open) set W such that

$$x \in W \subseteq (\text{resp., } p\text{-}w) \subseteq U.$$

Note (2.2): It means that a topological space (X, T) is a Hausdorff (resp., p -Hausdorff, s -Hausdorff) space iff every point of X is Hausdorff (resp., p -Hausdorff, s -Hausdorff) point.

Note (2.3): A topological space (X, T) (resp., p -regular) regular iff every point of X is a super regular (resp., super p -regular) point of X .

Remark (2.1): Every Hausdorff (resp., p -Hausdorff, Quasi- p -Hausdorff) point is a Frechet (resp., p -Frechet, Quasi-Frechet) point and a super Frechet point but the converse may not be true.

Example (2.1): Let (X, T) be a topological space where $X = \{a, b, c\}$, $T = \{\phi, \{a\}, \{b, c\}, X\}$. Then it is not a T_0 -space but the point a is a Hausdorff point.

Example (2.2): Let (X, T) be a topological space (X, T) where $X = \{a, b, c, d\}$

$$T = \{\phi, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}, X\}$$

$$(p\text{-}T) = \{\phi, \{a\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$$

and $(s\text{-}T) = \{\phi, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}, X\}$

Here b is a Frechet point but neither a super Frechet point nor a Hausdorff point. b, c , and d are p -Frechet points, c and d are p -super Frechet points.

Theorem (2.4): A closed subset A of a topological space (X, T) is super sub normal subset iff for every open set U containing A there exists an open set V such that

$$A \subseteq V \subseteq \bar{V} \subseteq U.$$

Proof: Let a closed set A is a super sub-normal set. Let U be an open set containing A . Then $U^C \cap A = \phi$ and U^C is closed. So there exists open sets V and W such that

$$A \subseteq V, U^C \subseteq W \text{ and } V \cap W = \phi$$

This means that

$$A \subseteq V \subseteq W^C \subseteq U.$$

Since W^C is closed, we have

$$A \subseteq V \subseteq \bar{V} \subseteq U.$$

This proves that the given condition is necessary.

Conversely, suppose that the given condition is satisfied for a closed set A . Let B be a closed set in X such that $A \cap B = \phi$. This means that B^C is an open set containing A . So according to the given condition there exist an open set V such that $A \subseteq V \subseteq \bar{V} \subseteq B^C$.

This gives

$$A \subseteq V, B \subseteq (\bar{V})^C \text{ and } V \cap \bar{V} = \phi$$

Hence A is a super sub-normal set.

Note (2.4): A topological space (X, T) is normal iff every closed subset of X is a super sub-normal set.

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