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A Brief Study of an Anisotropic Magneto-hydrodynamic Universe in General Relativity

by **Mantu Kumar,** *P.G. Department of Mathematics,*
M.S. College, Motihari-845401, India
(B.R.A Bihar University, Muzaffarpur-842001, India)
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Abstract:

I have tried to investigate an anisotropic cosmological model within General Relativity, featuring a magneto-hydrodynamics (MHD) Fluid by derive the evolution equations.

This work mainly depends on investigates a cosmological model of an anisotropic universe described by General Relativity, in corporating the effects of a homogeneous magnetic field using covariant magneto-hydrodynamics. I have explored exact solutions to the field equations, often using specific anisotropic metrics to analyze the evolution of shear, expansion and magnetic field generation. Here, we present a detailed mathematical equations study of an anisotropic universe governed by the laws of general relativity in the presence of a large scale magnetic field.

Keywords: Anisotropy, Magneto-hydrodynamics, General relativity, Cosmological models, methodology.

1. Introduction:

In recent years there has been a lot of interest in magneto-hydrodynamic cosmologies in general relativity. Cosmological models in the presence of a magnetic field have been studies by Zel'dovich (1965) and Thorne (1967).

Roy and Prakash (1978) taking the cylindrically symmetric metric of Marder (1958) has constructed a spatially homogeneous cosmological model in the presence of an incident magnetic field which is also anisotropic and non-degenerate Petrov type I. In this paper the energy momentum tensor has been assumed to be that of a perfect fluid with an element magnetic field and a spatially homogeneous cosmological model has been obtained. It is found that the model represents an expanding and shearing but non-rotating fluid which is also geodesic. I have also shown that the model has a 4-current which is either zero or space like. The latter corresponds to the case of magneto hydrodynamics. The requirement that the conductivity be positive impress an additional restriction on the metric potentials. It is found that the electromagnetic field gives positive contributions to the expansions, shear and free gravitational field which discount for large values of time at as lower rate than the corresponding quantities in the absence of the electromagnetic field. When the cosmological constraints amrit is found that in the absence of electromagnetic field pressure and density become equal and conversely if pressure and density are equal. There is no electromagnetic field.

2. Solutions of the field equations:

The cylindrically symmetric metric is considered here as

$$ds^2 = A^2(dt^2 - dx^2) - B^2dy^2 - C^2dz^2 \tag{2.1}$$

where A, B, C are functions of t only. The distribution consists of a perfect fluid and an electromagnetic field.

Thus

$$G_{ij} + \Delta g_{ij} = -K[(\rho + p)\lambda_i \lambda_j - pg_{ij} + E_{ij}] \tag{2.2}$$

$$g_{ij}\lambda_i \lambda_j = 1 \tag{2.3}$$

$$E_{ij} = g^{\alpha\beta} f_{i\alpha} F_{j\beta} - \frac{1}{4} g_{ij} F_{\alpha\beta} F^{\alpha\beta} \tag{2.4}$$

$$F_{[ij,k]} = 0 \tag{2.5}$$

$$F_j^{is} = J^i \tag{2.6}$$

Whole E_{ij} is the electromagnetic energy momentum tensor, F_{ij} is electromagnetic field tensor, Δ cosmological constant, J^i the current 4-vector and ρ and p are the density and pressure of the distribution. The coordinates are chosen to be comoving so that

$$\lambda^1 = \lambda^2 = \lambda^3 = 0, \quad \lambda^4 = {}^1/_A \tag{2.7}$$

1 Label the coordinates $(x, y, z, t) = (x^1, x^2, x^3, x^4)$

The off diagonal components of (2.2) are

$$\left.\begin{array}{ll}
\text{(a)} & F_{12}F_{24}B^{-2} + F_{12}F_{34}C^{-2} = 0 \\
\text{(b)} & F_{12}F_{14}A^{-2} - F_{23}F_{34}C^{-2} = 0 \\
\text{(c)} & F_{13}F_{14}A^{-2} + F_{23}F_{24}B^{-2} = 0 \\
\text{(d)} & F_{14}F_{24}A^{-2} - F_{13}F_{23}C^{-2} = 0 \\
\text{(e)} & F_{14}F_{34}A^{-2} + F_{12}F_{23}B^{-2} = 0 \\
\text{(f)} & F_{24}F_{34} - F_{12}F_{13} = 0
\end{array}\right] \tag{2.8}$$

Which lead to three possible cases:

(i) $F_{24} = F_{34} = F_{12} = F_{13} = 0$ at least one of F_{14}, F_{23} non-zero i.e. when the field F_{ij} in X-direction only.

(ii) $F_{14} = F_{34} = F_{12} = F_{23} = 0$ at least one of F_{24}, F_{13} non-zero i.e. when the field is in Y-direction only.

(iii) $F_{14} = F_{24} = F_{13} = F_{23} = 0$ at least one of F_{34}, F_{12} non-zero i.e. when the field is in Z-direction only.

Hence, the electromagnetic field is non-rules and consists of an electric and/or magnetic field both of which are in the direction of same space axis.

Without loss of generality only case I is considered in which the fields are in the X-direction L take

$$F_{14}^2 A^{-4} + F_{23}^2 B^{-2} C^{-2} = L^2 \tag{2.9}$$

The diogonal components of equation (2.2) may be written as

$$\frac{2}{A^2}\left[\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{A_4 B_4}{AB} - \frac{A_4^2}{A^2}\right]$$

$$-2\Delta = -K[-L^2 + (\rho + 3p)] \tag{2.10}$$

$$\frac{2}{A^2}\left[\frac{A_{44}}{A} + \frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{A_4^2}{A^2}\right] + 2\Delta = -K[L^2 + (\rho - p)] \tag{2.11}$$

$$\frac{-2}{A^2}\left[\frac{B_{44}}{B} + \frac{B_4 C_4}{BC}\right] + 2\Delta = -K[-L^2 + (\rho - p)] \tag{2.12}$$

$$\frac{-2}{A^2}\left[\frac{C_{44}}{C} + \frac{B_4 C_4}{BC}\right] + 2\Delta = -K[-L^2 + (\rho - p)] \tag{2.13}$$

Where suffi. four after the symbols A, B, C, stands for ordinary differentiation with respect to time. It is evident from these equations that L^2, ρ and p are each functions of time alone. From equation (2.5) and (2.9) it follows that F_{23} is a constant and F_{14} is a functions of t only i.e.

$$F_{23} = K, F_{14} = \pm A^2 (L^2 - K^2 B^{-2} e^{-2})^{½} \tag{2.14}$$

Where K is a constant the case when there is no electric field i.e. when $F_{14} = 0$. I have $J^i = 0$. It is the case considered by Roy and Prakash (1978). It is assumed that $F_{14} = 0$ and found the only non-zero component of J^i to be

$$J^i = \pm \frac{1}{A^2 BC} \frac{\delta}{\delta t} [BC(L^2 - K^2 B^{-2} C^{-2})^{½}] \tag{2.15}$$

Equation (2.15) shows that J^i is space like, unless $L^2 = f B^{-2} C^{-2}$, where f is constant in which case $J^i = 0$. The four current J^i is ingeneral the sum of the convection current and the conduction current.

$$J^i = \varepsilon_0 \lambda^i + \xi \lambda_j F^{ij} \tag{2.16}$$

where ε_0 is the rest charge density and ξ in the conductivity. In the case considered here, I have $\varepsilon_0 = 0$ i.e. magnetic hydrodynamic. Thus

$$J^i = -\frac{1}{A} I_4, I^{-1} \tag{2.17}$$

where $I = BC(L^2 - K^2 B^{-2} C^{-2})^{1/2}$

The requirement of positive conductivity in (2.17) parts further restriction on A, B, C. Hence in magneto hydrodynamic case metric potentials are restricted not only by the field equations and energy conditions (Howking and Penrose 1973) they are also restricted by the requirement that the conductivity become for a realistic model.

Equations (2.10) - (2.13) are four equations in six unknowns A, B, C, ρ, p and L. For complete determinacy of this system of equations, two assumptions are made.

(i) F^{14} is such that

$$L^2 = l^2 B^{-4} C^{-4}, \text{ where } l \text{ is constant.} \tag{2.18}$$

(ii) The space time is Petrov type - I degenerate (the degeneracy being in y and z directions) which requires that

$$C^{12}_{12} = C^{13}_{13} \text{ with } B \neq C \tag{2.19}$$

This from (2.19), I have

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{2A_4}{A}\left(\frac{C_4}{C} - \frac{B_4}{B}\right) = 0 \tag{2.20}$$

Equations (2.12) and (2.13) yield

$$\frac{B_{44}}{B} = \frac{C_{44}}{C} \tag{2.21}$$

Which on integrations gives

$$B_4 C - BC_4 = K_1 \tag{2.22}$$

K_1 being on arbitrary constant.

Further from (2.20) and (2.21), we get

$$\frac{A_4}{A}\left(\frac{C_4}{C} - \frac{B_4}{B}\right) = 0. \tag{2.23}$$

Since $B \neq C$, eqn. (2.23) gives

$$A = N \text{ (a constant)}. \tag{2.24}$$

From eqns. (2.11), (2.12) and (2.24) we have

$$\frac{B_{44}}{B} + \frac{B_4 C_4}{BC} = -KN^2 L^2. \tag{2.25}$$

Putting $B/C = \alpha$ and $BC = \beta$, eqn. (2.22) reduces to

$$\left(\frac{\alpha_4}{\alpha}\right)\beta = K_1 \tag{2.26}$$

and eqn. (2.25) turns into

$$\frac{1}{\beta}\left[\left(\frac{\alpha_4}{\alpha} + \frac{\beta_4}{\beta}\right)\beta\right]_4 = -2KN^2 L^2. \tag{2.27}$$

From eqns. (2.26) and (2.27) we have

$$\frac{\beta_{44}}{\beta} = -2KN^2 L^2 \tag{2.28}$$

which after the use of eqn. (2.18) goes to the form

$$\beta_{44} = -\frac{2KN^2 l^2}{\beta^3}. \tag{2.29}$$

Equation (2.29) on integration, gives

$$[\beta_4]^2 = \frac{2KN^2 l^2}{\beta^2} + k_2^2 \tag{2.30}$$

where k_2^2 is an arbitrary constant.

From eqns. (2.26) and (2.30), we get

$$\frac{d\alpha}{\alpha} = \frac{k_1}{k_2}\frac{d\beta}{(\beta^2 + k_3^2)^{1/2}} \tag{2.31}$$

where

$$k_3^2 = \frac{2KN^2l^2}{k_2^2}. \tag{2.32}$$

Integration of eqn. (2.31) gives

$$\alpha = k_4 \left[\beta + \left(\beta^2 + k_3^2\right)^{1/2} \right]^{k_1/k_2} \tag{2.33}$$

k_4 being a constant of integration. Therefore

$$B^2 = k_4 \beta \left[\beta + \left(\beta^2 + k_2^2\right)^{1/2} \right]^{k_1/k_2} \tag{2.34}$$

and

$$C^2 = \frac{\beta}{k_4} \left[\beta + \left(\beta^2 + k_3^2\right)^{1/2} \right]^{-k_1/k_2} \tag{2.35}$$

Hence the metric (2.1) can be written as

$$ds^2 = A^2 \left[\frac{d\beta^2}{(d\beta/dt)^2} - dx^2 \right] - B^2 dy^2 - C^2 dz^2 \tag{2.36}$$

which by use of eqns. (2.24), (2.30), (2.34) and (2.35) takes the form

$$\begin{aligned} ds^2 = N^2 & \left[- dx^2 + \frac{d\beta^2}{(k_2^2/\beta^2)(\beta^2 + k_3^2)} \right] \\ & - k_4 \beta \left[\beta + \left(\beta^2 + k_3^2\right)^{1/2} \right]^{k_1/k_2} dy^2 \\ & - \frac{\beta}{k_4} \left[\beta + \left(\beta^2 + k_3^2\right)^{1/2} \right]^{-k_1/k_2} dz^2 \end{aligned} \tag{2.37}$$

The transformation

$$Nx \to X,\ k_4 y \to Y,\ k_4^{-1} z \to Z,\ \beta \to \sqrt{(T^2 - k_3^2)} \tag{2.38}$$

reduces (2.37) to the form

$$\begin{aligned} ds^2 = \frac{N^2 dT^2}{k_2^2} - dX^2 & - \left(T^2 - k_3^2\right)^{1/2} \left[T + \left(T^2 - k_3^2\right)^{1/2} \right]^{k_1/k_2} dY^2 \\ & - \left(T^2 - k_3^2\right)^{1/2} \left[T + \left(T^2 - k_3^2\right)^{1/2} \right]^{-k_1/k_2} dZ^2 \end{aligned} \tag{2.39}$$

which can be further transformed to the metric

$$ds^2 = dT^2 - dX^2 - \left(T^2 - P^2\right)^{1/2}\left[T + \left(T^2 - P^2\right)^{1/2}\right]^3 dY^2$$
$$- \left(T^2 - P^2\right)^{1/2}\left[T + \left(T^2 - P^2\right)^{1/2}\right]^{-3} dZ^2 \quad (2.40)$$

This metric has no singularity and will be real only when $T^2 > P^2$.

3. Some Physical Features:

(a) The Distribution in the Model:

For the model (40) pressure p and density ρ are given by

$$K_p = \frac{T^2}{4}(T^2 - P^2)^{-2} - \frac{q^2}{4}(T^2 - P^2)^{-1} + \frac{3Kl^2}{2}(T^2 - P^2)^{-2} + \Lambda \quad (3.1)$$

$$K_\rho = \frac{T^2}{4}(T^2 - P^2)^{-2} - \frac{q^2}{4}(T^2 - P^2)^{-1} + \frac{Kl^2}{2}(T^2 - P^2)^{-2} - \Lambda \quad (3.2)$$

The model has to satisfy the reality conditions (Ellis 1971)

(i) $\rho + p > 0$

(ii) $\rho + 3p > 0$

which requires that

$$P^2 < T^2 < \frac{P^2 q^2 + 4Kl^2}{(1 - q^2)} \quad (3.3)$$

and

$$\Lambda > \frac{-1}{2(T^2 - P^2)^2}\left[(1 - q^2)\, T^2 + P^2 q^2 + 5Kl^2\right] \quad (3.4)$$

The condition (3.3) holds only when $q^2 < 1$.

In the case of disordered radiation ($\rho = 3p$) we have

$$\Lambda = \frac{-1}{8(T^2 - P^2)^2}\left[(1 - q^2)\, T^2 + P^2 q^2 + 8Kl^2\right] \quad (3.5)$$

and

$$K_\rho = 3K_p = \frac{1}{8(T^2 - P^2)^2}\,[5(1 - q^2)\,T^2 + 5P^2q^2 + 4Kl^2] \tag{3.6}$$

and in the case of stiff matter ($\rho = p$)

$$\Lambda = \frac{-Kl^2}{2(T^2 - P^2)^2} \tag{3.7}$$

and

$$K_p = K_\rho = \left[\frac{T^2}{4}(T^2 - P^2)^{-2} - \frac{q^2}{4}(T^2 - P^2)^{-1}\right] + \frac{Kl^2}{2}(T^2 - P^2)^{-2} \tag{3.8}$$

The flow vector λ^i is given by

$$\lambda^1 = \lambda^2 = \lambda^3 = 0,\ \lambda^4 = 1 \tag{3.9}$$

The flow vector λ^i satisfies $\lambda^i_{ij}\lambda^i = 0$. Thus the lines of flow are geodesics. Tensor of rotation W_{ij} defined by

$$W_{ij} = \lambda_{i;j} - \lambda_{j;i} \tag{3.10}$$

is identically zero. Thus the fluid filling the universe is non-rotational.

The scalar of expansion $\Theta = \lambda^i_{ji}$ is given by

$$\Theta = \frac{T}{(T^2 - P^2)^{3/2}} \tag{3.11}$$

which tends to zero when $T \to \infty$.

The components of the shear tensor defined by

$$\sigma_{ii} = \frac{1}{2}\left(\lambda_{i;j} + \lambda_{j;i}\right) - \frac{1}{3}\Theta\left(g_{ij} - \lambda_i\lambda_j\right) \tag{3.12}$$

are

$$\sigma_{11} = \frac{1}{3T}\left(T^2 - P^2\right)^{-3/2},$$

$$\sigma_{22} = \left[T + \left(T^2 - P^2\right)^{1/2}\right]^q \left\{-\frac{1}{2}\left[T\left(T^2 - P^2\right)^{-1/2} + q\right] + \frac{1}{3}T\left(T^2 - P^2\right)^{-1}\right\},$$

$$\sigma_{33} = \left[T + \left(T^2 - P^2\right)^{1/2}\right]^{-q} \left\{-\frac{1}{2}\left[T\left(T^2 - P^2\right)^{-1/2} - q\right] + \frac{1}{3}T\left(T^2 - P^2\right)^{-1}\right\},$$

$$\sigma_{44} = 0; \tag{3.13}$$

the other components of σ_{ij} being zero.

The non-vanishing components of the conformal curvature tensor C^{hi}_{jk} are

$$C^{12}_{12} = C^{13}_{13} = -\frac{1}{2}C^{23}_{23} = -\frac{1}{6}\left[\frac{1}{2}\left(T^2 - P^2\right)^{-1/2} - 3T^2\left(T^2 - P^2\right)^{-3/2} + \frac{1}{2}q^2\left(T^2 - P^2\right)^{-1} - \frac{1}{4}T^2\left(T^2 - P^2\right)^{-2}\right] \tag{3.14}$$

The non-vanishing component of the charge current 4-vectors is given by

$$J^1 = l^2 T\left(T^2 - P^2\right)^{-2}\left[l^2 - k^2\left(T^2 - P^2\right)\right]^{-1/2} \tag{3.15}$$

The conductivity is given by

$$\zeta = l^2 T\left(T^2 - P^2\right)^{-1}\left[l^2 - k^2\left(T^2 - P^2\right)\right]^{-1} \tag{3.16}$$

For a physically realistic MHD model ζ has to be positive which requires that

$$0 < T < \left(k^2 P^2 + l^2\right)^{1/2} / k.$$

(b) The Doppler Effect in the Model:

The track of a light pulse in the model (2.40) is obtained by setting

$ds^2 = 0$ i.e.

$$\left(\frac{dX}{dT}\right)^2 + \left(T^2 - P^2\right)^{1/2}\left[T + \left(T^2 - P^2\right)^{1/2}\right]^q \left(\frac{dY}{dT}\right)^2 + \left(T^2 - P^2\right)^{1/2}\left[T + \left(T^2 - P^2\right)^{1/2}\right]^{-q}\left(\frac{dZ}{dT}\right)^2 = 1 \tag{3.17}$$

For the case when velocity is along z-axis, eqn. (3.17) gives

$$\frac{dZ}{dT} = \pm\left(T^2 - P^2\right)^{-1/4}\left[T + \left(T^2 - P^2\right)^{1/2}\right]^{q/2}$$

$$= \pm\,\psi(T). \tag{3.18}$$

Hence the light pulse leaving a particle at (0, 0, z) at time T_1 would arrive at a later time T_2 given by

$$\int_{T_1}^{T_2} \psi(T)\, dT = \int_0^Z dZ \tag{3.19}$$

Hence

$$\psi_2(T)\, \delta T_2 = \psi_1(T)\, \delta T_1 + \frac{dZ}{dT} \delta T_1$$

$$= \psi_1(T)\, \delta T_1 + U_z \delta T_1 \tag{3.20}$$

where $(dZ/dT) = U_z$ is the z-component of the velocity of the particle at the time of emission and $\psi_1(T)$ and $\psi_2(T)$ are the values of $\psi(T)$ for $T = T_1$ and $T = T_2$ respectively. From the above equation we get

$$\delta T_2 = \left\{ \frac{\psi_1(T) + U_z}{\psi_2(T)} \right\} \delta T_1 \tag{3.21}$$

The proper time interval δT_1^0 between successive wave crests as measured by the local observer moving with the source is given by

$$\delta T_1^0 = \left\{ 1 - \left(\frac{dX}{dT}\right)^2 - \left(T^2 - P^2\right)^{1/2} \left[T + \left(T^2 - P^2\right)^{1/2} \right]^q \left(\frac{dY}{dT}\right)^2 \right.$$

$$\left. - \left(T^2 - P^2\right)^{1/2} \left[T + \left(T^2 - P^2\right)^{1/2} \right]^{-q} \left(\frac{dZ}{dT}\right)^2 \right\}^{1/2} \delta T_1 \tag{3.22}$$

This can be written as

$$\delta T_1^0 = \{1 - U^2\}^{1/2}\, \delta T_1 \tag{3.23}$$

where U is the velocity of the source at the time of emission. Similarly we may write

$$\delta T_2^0 = \delta T_2 \tag{3.24}$$

as the proper time interval between the reception of two successive wave crests by an observer at rest at the origin. Hence following Tolman (1962), the red shift in this case is given by

$$\frac{\lambda+\delta\lambda}{\lambda}=\frac{\delta T_2^0}{\delta T_1^0}$$

$$=\frac{\left\{\left(T_1^2-P^2\right)^{-1/4}\left[T_1+\left(T_1^2-P^2\right)^{1/2}\right]^{q/2}+U_z\right\}}{\left\{\left(T_2^2-P^2\right)^{-1/4}\left[T_2+\left(T_2^2-P^2\right)^{1/2}\right]^{q/2}\right\}\left\{1-U^2\right\}^{1/2}} \tag{3.25}$$

(c) Newtonian Analogue of Force in the Model:

Here we study the effect of electromagnetic field in the force terms R_i and S_i (Narlikar and Singh 1951). The vector R_i and S_i are defined as follows (Narlikar and Singh 1951):

$$R_i=\Delta^i_{ji}=H,_i/H \tag{3.26}$$

$$S_i=\Delta^l_{jk}\,g^{jk}g_{li}$$

$$=g^{jk}g_{ji,k}-H,_i/H \tag{3.27}$$

where

$$H=\sqrt{g/\gamma}$$

For the line element (2.40) we have

$$\left.\begin{aligned} g_{11}&=-1\\ g_{22}&=-\left(T^2-P^2\right)^{1/2}\left[T+\left(T^2-P^2\right)^{1/2}\right]^{q}\\ g_{33}&=-\left(T^2-P^2\right)^{1/2}\left[T+\left(T^2-P^2\right)^{1/2}\right]^{-q}\\ g_{44}&=1\end{aligned}\right\} \tag{3.28}$$

and

$$\left.\begin{aligned} g^{11}&=-1\\ g^{22}&=-\left(T^2-P^2\right)^{-1/2}\left[T+\left(T^2-P^2\right)^{1/2}\right]^{-q}\\ g^{33}&=-\left(T^2-P^2\right)^{-1/2}\left[T+\left(T^2-P^2\right)^{1/2}\right]^{q}\\ g^{44}&=1\end{aligned}\right\} \tag{3.29}$$

$$g = -\left(T^2 - P^2\right) \tag{3.30}$$

The corresponding flat metric $\gamma_{\mu\nu}$ is taken to be that of special relativity

$$ds^2 = dT^2 - dX^2 - dY^2 - dZ^2 \tag{3.31}$$

Thus

$$\gamma_{ij} = [-1, -1, -1, +1] \tag{3.32}$$

and

$$\gamma = -1 \tag{3.33}$$

From (3.30) and (3.33)

$$H = \sqrt{g/\gamma} = \left(T^2 - P^2\right)^{1/2} \tag{3.34}$$

From (3.26) and (3.27) we get

$$R_i = \left[0, 0, 0, T\left(T^2 - P^2\right)^{-1}\right] \tag{3.35}$$

and

$$S_i = \left[0, 0, 0, -T\left(T^2 - P^2\right)^{-1}\right] \tag{3.36}$$

Thus we find that Newtonian analogue of R_i and S_i both are null force vectors. R_4 and S_4 have no Newtonian analogues.

In the absence of electromagnetic field the model is given by the metric

$$ds^2 = dT^2 - dX^2 - \frac{1}{2}(2T)^{q+1}\, dY^2 - \frac{1}{2}(2T)^{-q+1}\, dZ^2 \tag{3.37}$$

for which the pressure p_0 and density δ_0 are given by

$$Kp_0 = \frac{1 - q^2}{4T^2} + \Lambda \tag{3.38}$$

$$K\rho_0 = \frac{1 - q^2}{4T^2} - \Lambda \tag{3.39}$$

The reality conditions (Ellis 1971) require that

$$q^2 < 1 \text{ and } \Lambda > \frac{q^2 - 1}{2T^2} \tag{3.40}$$

Therefore vectors R_i and S_i reduce to

$$R_i = \left[0, 0, 0, \frac{1}{T}\right] \tag{3.41}$$

and

$$S_i = \left[0, 0, 0, -\frac{1}{T}\right] \tag{3.42}$$

The flow vector λ^i satisfies $\lambda^i_{;j}\lambda^j = 0$. Thus the lines of flow are geodesics. The tensor of rotation is identically zero. The scalar of expansion is given by

$$\Theta = 1/T^2 \tag{3.43}$$

The non-zero components of the shear tensor are

$$\left.\begin{aligned} \sigma_{11} &= \frac{1}{3T^4} \\ \sigma_{22} &= \frac{1}{3}(2T)^{q-1}\,[-3T(1+q)+2] \\ \sigma_{33} &= \frac{1}{3}(2T)^{-q-1}\,[-3T(1-q)+2] \end{aligned}\right\} \tag{3.44}$$

The red shift in the model is given by

$$\frac{\lambda + \delta\lambda}{\lambda} = \frac{\left[(2)^{q/2}\, T_1^{(q-1)/2} + U_z\right]}{\left[(2)^{q/2}\, T_2^{(q-1)/2}\right]\left(1 - U^2\right)^{1/2}} \tag{3.45}$$

where

$$U_z = \frac{dZ}{dT} = (2)^{q/2}\,(T)^{(q-1)/2}$$

is the velocity at the time of emission.

The non-vanishing components of the conformal curvature tensor are

$$C^{12}_{12} = C^{13}_{13} = -\frac{1}{2}C^{23}_{23} = -\frac{1}{24}\left[-\frac{10}{T} + \frac{(2q^2-1)}{T^2}\right] \tag{3.46}$$

As $T \to \infty$, shear, expansion and free gravitational fields vanish.

Thus, we get by these above equations the electromagnetic field gives positive contributions to expansion, shear and free gravitational field which die out for large values of T at a slower rate than the corresponding quantities in the absence of the electromagnetic field.

4. Conclusion:

In this paper, we get that the studied anisotropic MHD model provides a self consistent solution within general relativity, describle the physical conditions under which the model is viable and discusses how its predictions either align with or diverge from current cosmological observations, often emphasizing that any large-scale anisotropy in the real universe must be minimal.

This study underscores the role of magnetic fields in shaping early cosmic evolution and demonstrates that anisotropic MHD model are valuable in understanding the transition from a potentially anisotropic early universe to the highly isotropic universe, we observe today.

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