Applied Science Periodical
ISSN 0972-5504
Volume - XXVI, No. 1, February 2024
Journal website: www.internationaljournalsiwan.com
ORCID Link: https://orcid.org/0009-0008-5249-8441
Google Scholar: https://scholar.google.com/citations?user=BRweiDcAAAAJ&hl=en
Refereed and Peer-Reviewed Quarterly Periodical

Construction of Linear O.R. models using Reverse Lexicographic Order

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(Received: January 02, 2024; Accepted: January 24, 2024;
Published Online: February 28, 2024)

Abstract:

This paper deals with the formulation of linear mathematical models by considering all partitions in reverse lexicographic order. Such linear O.R. models are formulated. It is observed that the solution to these models is all integer valued solution.

Keywords: Operations Research (O.R.), Integer Programming Problem (I.P.P.), Linear Programming Problem (L.P.P.), reverse lexicographic order, TORA package.

Introduction:

Integer programming is a valuable tool in O.R. having tremendous applications in business and industry. Integer programming problem is a mathematical programming technique that produces optimum integer solutions to a L.P.P.

Main areas in which IPP is used are: preparation of time table, capital budgeting, depot location, job shop scheduling, assembly line balancing, imposition of logical conditions in LPPs, airline crew scheduling, production scheduling, all allocation problems involving men and machine.

Construction of decomposition matrix using reverse lexicographic order:

A partition of I into P parts such that $1 \leq P \leq I$ is any of the partitions of I having exactly P parts. For any given I and for all P such that

$1 \leq P_1 \leq P \leq P_2 \leq 1$, we write all distinct length P decompositions of I.

Hence for $I = 6$, $P_1 = 1$, $P_2 = 6$, the decompositions of I into I summands i.e. all partitions of I in reverse lexicographic order is as follows:

6					
5	1				
4	2				
3	3				
4	1	1			
3	2	1			
2	2	2			
3	1	1	1		
2	2	1	1		
2	1	1	1	1	
1	1	1	1	1	1

Theorem: If we construct a L.P.P. with n decision variables such that $[A]$ is a decomposition matrix as mentioned above, $[c]$ and $[b]$ consists of first n natural numbers then the optimum solution is obtained through two iterations and is given by $x_n = b_i$ where $a_{ij} = 1$ for $j = 1, 2, \ldots\ldots, n$. For the dual also, the solution is obtained in two iterations and is given by $x_n = b_i$ where $a_{ij} = 1$ for $j = n$ and $a_{ij} = 0$ for $j \neq n$. While constructing such LPPs, it should be noted that each permutation is considered only once.

Illustration:

Consider a maximization type of LPP such that $[A]$ is the decomposition matrix as mentioned above, $[b] = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11)'$ and $[c] = (1, 2, 3, 4, 5, 6)$ and x_i are non-negative integers.

We solve the above example using TORA package and the optimum solution summary is as follows:

Final iteration No. : 2

Objective value(max) = 66

Variable	Value	Obj. Coeff.	Obj. Val. Contrib.
X_1	0	1	0
X_2	0	2	0
X_3	0	3	0
X_4	0	4	0
X_5	0	5	0
X_6	11	6	66

Constraint	R.H.S.	Slack-/Surplus+
1(<)	1	1.00-
2(<)	2	2.00-
3(<)	3	3.00-
4(<)	4	4.00-
5(<)	5	5.00-
6(<)	6	6.00-
7(<)	7	7.00-
8(<)	8	8.00-
9(<)	9	9.00-
10(<)	10	10.00-
11(<)	11	0.00

Sensitivity analysis

Variable	Current obj. coeff.	Min obj. coeff.	Max obj. coeff.	Reduced cost
X_1	1	-infinity	6	5
X_2	2	-infinity	6	4
X_3	3	-infinity	6	3
X_4	4	-infinity	6	2
X_5	5	-infinity	6	6
X_6	6	4	infinity	0

Constraint	Current RHS	Min RHS	Max RHS	Dual Price
1(<)	1	0	∞	0
2(<)	2	0	∞	0
3(<)	3	0	∞	0
4(<)	4	0	∞	0
5(<)	5	0	∞	0
6(<)	6	0	∞	0
7(<)	7	0	∞	0
8(<)	8	0	∞	0
9(<)	9	0	∞	0
10(<)	10	0	∞	0
11(<)	11	0	∞	6

Remarks: Hence the formulation of such linear O.R. models by considering reverse lexicographic order gives us interesting results such as:

(i) We obtain the optimum solution in just two iterations.

(ii) The optimum solution is all integer valued solution.

(iii) Such primal and its dual also have all integer valued solution.

(iv) It is also observed that the optimum value of objective function is same as the sum of elements in [*A*].

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