The Mathematics Education **ISSN 0047-6269**
Volume - LV, No. 3, September 2021
Refereed and Peer-Reviewed Quarterly Journal
Journal website : www.internationaljournalsiwan.com

A Brief Study of Contributions of Ancient Indian Mathematician Bhāskarāchārya

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Abstract :

In this paper we deal with the contributions of Ancient Indian Mathematician Bhāskarāchārya. Who wrote Siddhanta Shiromani in the year 1150 A.D. at the age of thirty six. The book is divided into four parts- Līlāvatī, Bijaganita, Goladhyaya, Grahaganita. He had ideas of Pell's equation, differential calculus etc. much before European Mathematicians.

Keywords : Bhāskarāchārya, Līlāvatī, Bijaganita, Grahaganita, Pell's equation.

1. Introduction :

Ancient Indian Saints and Scientists have made great contributions to Mathematics. The period has been divided into several sections from 400 A.D.-1600 A.D. when a galaxy of scholars such as Aryabhata, Brahmgupta, etc. made contributions. Before this there was vedic period, post-vedic period, Jain Mathematics, etc. We have considered Bhāskarāchārya who wrote Siddhanta Shiromani at the age of thirty six. The book is divided into four chapters comprising Līlāvatī, Bijaganita, Grahaganita, etc.

2. Bhāskarāchārya :

Bhāskarāchārya is one of the most well-know mathematicians of ancient India. He was born in a village Bijaragi of Vijayapura (Bijapur) in Karnataka in a Brahmin family in 1114 A.D. History records that his grandfather was holding a hereditary post as a court scholar, as well as his son and other descendants. His father Mahesvara was a mathematician and astronomer who taught him mathematics. He later taught mathematics to his son Loksamudra. He deid in 1185 A.D.

He wrote Siddhant Shiromani in the year 1150 A.D. when he was thirty six years old. This book is divided into four parts, with the names *Līlāvatī, Bijaganita, Goladhyaya and Grahaganita*, Līlāvatī and Bijaganita deal with arithmetic and algebra respectively while Goladhyaya and Grahaganita relate to astronomy.

2.1 Līlāvatī :

Līlāvatī, named after his daughter consists of 278 verses. It covers calculations, progression, measurement, permutations and other topics.

2.2 Bijaganita :

Its second edition has 213 verses. It deals with zero positive and negative numbers infinity and indeterminate equations which are now called Pell's equation using a Kuttak method. He also solved $61x^2 + 1 = y^2$ which was done by European mathematicians, such as Euler centuries later.

2.3 Grahaganita :

This section investigated motion of planets including their instantaneous speeds. It consists of 451 verses. He arrived at the approximation

$$\sin y' - \sin y \approx (y' - y)\cos y$$

when y' is very close to y. In modern calculus it is stated as

$$\frac{d}{dy}(\sin y) = \cos y$$

He stated that at its highest point a planets instantaneous speed is zero.

Some of Bhaskara's contribution to mathematics include the following-

He calculated the same area in two different ways and cancelling out terms, he divided $a^2 + b^2 = c^2$ i.e. that Pythagorean theorem.

In Līlāvatī, the explained solutions of quadratic cubic and quadratic indeterminate equations. For solving indeterminate equation of the type $ax^2 + bx + c = y$ he used cyclic cakravāla method. The solution to this equation was later found by William Brouncker in 1657, but his method was more difficult than the cakravāla method. He gave preliminary concept of mathematical analysis he stated Rolle's theorem which is a special case of a very important theorem in analysis, the mean value theorem.

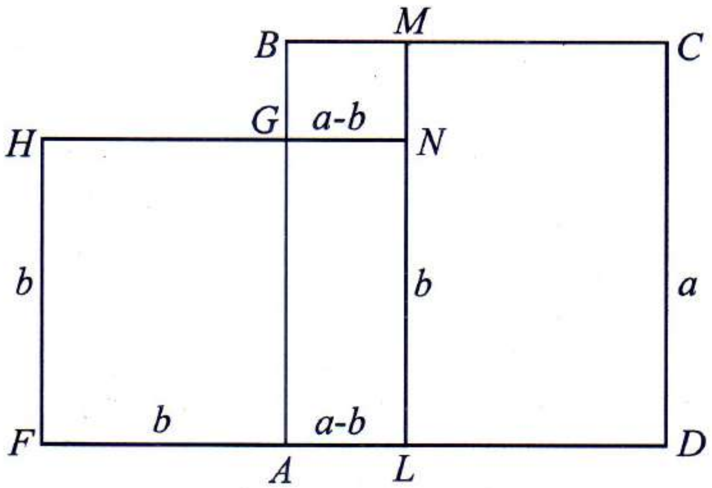
He developed spherical trigonometry.

He gave an estimation of π.

He treated surds and quadratic equations.

He was aware that when a variable attains the maximum value, it differential is O.

He has proved some algebraic relations with help of geometry.



Here $ABCD$ is a square with each side a. Let $FAGH$ be a square with each side b. Take $AL = a-b$. LM is perpendicular to AD and let $LN = b$. $ADCB$ is square with each side a.

Then $MN = LM - LN = a - b = AL = GN$

Thus $GNMB$ is a square with each side $a-b$.

Now $HN = FL = FA + AL = b + a - b = a$

This we write as

$$(a-b) + b = a$$

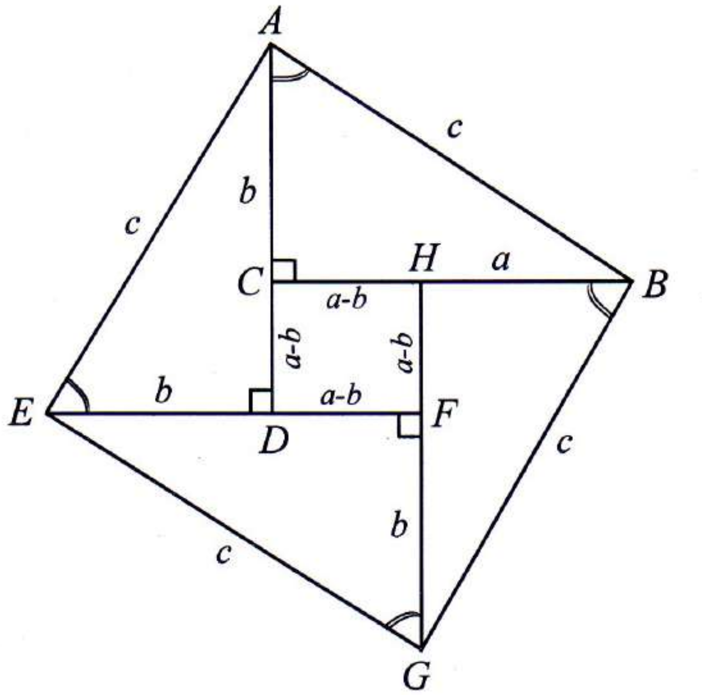
Also $FA = FL - AL$

$$\Rightarrow \quad b = a - (a-b)$$

Thus, we have proved geometrically

$$a - (a-b) = b.$$

Now, we come to Bhaskara's proof of Pythagorean theorem.



We consider right-angled triangle ABC where $AC = b$, $BC = a$, $AB = c$.

Let $a > b$. Then $a - b > 0$.

We want to show $c^2 = a^2 + b^2$

For this we do some construction. Produce AC to D such that $AD = a$.

Then $CD = AD - AC = a - b$

Let EF be a line perpendicular to AD such that $ED = b$ and $DF = a - b$

Let $\angle CAB = \theta$, then $\tan\theta = \frac{a}{b}$

In ΔEAD, $\tan AED = \frac{a}{b}\cdot$ Hence $\angle AED = \theta$.

So $\angle EAD = \frac{\pi}{2} - \theta$.

Hence $\angle EAB = \angle EAD + \angle CAB = \frac{\pi}{2} - \theta + \theta = \frac{\pi}{2}$

So $EA \perp AB$ and since $\Delta S\ ACB$ and ADE are similar.

$$AE = AB = C$$

Now $CD = a - b = DF$. We complete square $DFHC$.

Produce HF to G such that $FG = b$.

Then EFG is a right-angled triangle such that

$$EF = ED + DF = b + a - b = a \text{ and } FG = b$$

So ΔEFG is similar to ΔADE and consequently as before,

$$EG = c \text{ and } \angle AEG = \frac{\pi}{2}\cdot$$

Now consider ΔHBG.

$$BH = BC - CH = a - (a - b) = b.$$

$$HG = HF + FG = a - b + b = a.$$

Thus ΔBHG is a right angled triangle similar to ΔEFG

Consequently $BG = C$ and $\angle EGB = \frac{\pi}{2}\cdot$

Also $\angle ABG = \frac{\pi}{2}\cdot$

Hence $AEGB$ is a square whose each side is c.

Hence area $AEGB = C^2$.

Now Δ^s ABC, AED, EFG and BHG are all similar area of each $= \frac{1}{2}ab$

area of square $CDFH = (a - b)^2$

$$\text{So area } AEGB = 4 \times \frac{1}{2}ab + (a-b)^2$$

$$= 2ab + a^2 + b^2 - 2ab$$

$$= a^2 + b^2$$

We have computed area of square $AEGB$ in two ways and hence they must be equal.

$$\text{So } c^2 = a^2 + b^2$$

Mughal emperor Akbar commanded his reputed minister Abul Fazal and Vatta Ulla Rushdie to translate the Līlāvatī and the Bijaganita into persian which were published in the reign of Jahangir. Its english translation was done by Henry Thomas Colebrooke.

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