ISSN 0972-5504

VOLUME - XXIII, No. 2, MAY 2021

APPLIED SCIENCE PERIODICAL

Since 1999

A Refereed and Peer-Reviewed Quarterly Periodical Devoted To Mathematical and Natural Sciences

Applied Science Periodical **ISSN 0972-5504**
Volume - XXIII, No. 2, May 2021
Refereed and Peer-Reviewed Quarterly Periodical
Journal website : www.internationaljournalsiwan.com

Analysis of Discrete Mathematics for Group Colouring

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Abstract :

Graph theory is a crucial topic in the field of discrete mathematics and graph colouring or group labeling is an important branch of graph theory, which can easily solve many real life problems. There are many real world problems that can be solved by graph theory and some real life problems exists where graph theory is the only alternative to solve these type of problems.

The main objective of this paper is to present the significance of group theory in our daily life. There are many real world problems that can be solved by graph theory and some real life problems exists where graph theory is the only alternative to solve these type of problems.

Introduction :

Discrete mathematics is an essential branch of applied mathematics and graph theory is a crucial topic in the field of discrete mathematics. Graph Colouring is a significant problem in graph theory. This problem is widely need to solve many real problems, viz; scheduling, resource allocation, traffic phasing, task assignment, etc. [1-5]. $L(h_1, h_2,......, h_n)$-labeling of the graph is the generalization of Graph Colouring problem and it has been widely applied to solve frequency assignment problem in wireless communication.

In this paper, we have shown some direct applications of discrete mathematics like applications of graph theory to mathematics like applications of graph theory to scheduling problems traffic signal lights, social networks and aircraft scheduling, etc.

Formulation :

Mathematics is classified into two branches, Applied mathematics and Pure mathematics. Groups are significant, since the graph is the only way of revealing information in pictorial form. We defined some terms related to graph theory most readily, such as

Definition 1.1 :

A graph $G = (V, E)$, where $V = \{v_1, v_2,......., v_n\}$ is called a path, denoted by P_n, if and only if $(v_i, v_{i+1}) \in E$, where $1 \leq i \leq n \leq -1$.

A path with 7 vertices is shown in Figure 1.

Figure 1 : A path with 7 vertices

Definition 1.2 :

A graph G, where $G = (V, E)$, where $V = \{v_1, v_2,......., v_n\}$ is called a cycle, denoted by C_n, if only $(v_i, v_{i+1}) \in E$, where $1 \leq i \leq n \leq 1$ and $(v_1, v_n) \in E$.

A cycle with 6 vertices is shown below.

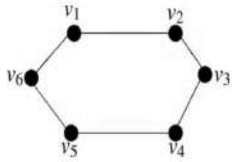

Figure 2 : A cycle with 6 vertices

Definition 1.3 :

An Euler path in a graph G is defined by a walk, which passes through each vertex of G and traverses every edge of G exactly once.

Definition 1.4 :

A simple undirected graph $G = (V, E)$ is called complete if each pair of vertices of G are adjacent. A complete graph with n vertices is denoted by K_n. That is $(u, v) \in E$ for all $u, v \in V$.

The Figure 3 presents a complete graph with 5 vertices

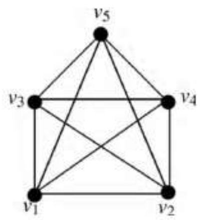

Figure 3 : A complete graph with 5 vertices K_5

Definition 1.5 :

A simple undirected graph $G = (V, E)$ is called complete bipartite graph if the vertex set V can be partitioned into two non empty subsets V_1 and V_2 so that there is no such edge between any two vertices in V_1 and also there is no such edge between any two vertices in V_2 but every vertex in V_1 is adjacent to every vertex in V_2. A complete bipartite graph with $p + q$ vertices is denoted by $K_{p,q}$. Obviously $|V_1| + |V_2| = p + q$.

A complete bipartite graph is given in Figure 4.

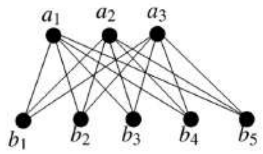

Figure 4 : A complete bipartite graph $K_{3,5}$

Graph Colouring :

Let $G = (V, E)$ be a simple graph. Graph Colouring, especially Vertex Colouring is the assignment of colours to the vertices of the graph so that no two adjacent vertices share the same colour. Therefore any graph having n vertices can easily be coloured using n colours (viz. one colour to one vertex). But, the main objective to colour a graph is to minimizing the number of colour used. The formal definition is presented below.

Definition 1.6 :

For any graph $G = (V, E)$ is a graph the proper colouring means, colour the vertices of G using least number of colours in such a way that adjacent vertices have different colour.

If k colours are required to proper colour a graph G then the graph G is called k-colourable. The least k for which G is k-colourable is called chromatic number and is denoted by $c(G)$. Conversely, a graph G is k-chromatic if $c(G) = k$. A proper k-colouring of a k-chromatic graph is an optimal colouring.

If a graph G is k-chromatic, then to colour all the vertices of G needs k colours, it cannot be coloured using k - 1 colours or less colours. Let us consider a graph with six vertices a, b, c, d, e and f (Shown in).

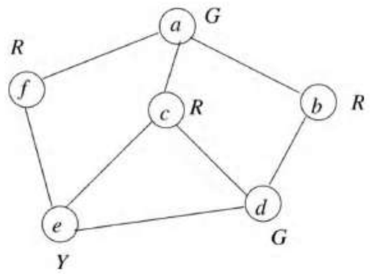

Figure 5 : A three colourable graph

First, we colour the vertex a by an arbitrary colour say G (green). Since, the vertex a is adjacent to the vertices b, c and f so, the colour of these vertices can never be green. So we colour these vertices by R(red). Note that the vertices b, c, f are not connected to each other. Now the colour of the vertex d can never be R(red) as it is adjacent to both the vertices b and c whose colour is R(red).

But this vertex can be coloured by G. The vertex e is adjacent to the vertices d, c, f and they are coloured by either G or R. So, to colour the vertex e we need a new colour say Y(yellow). There is no vertices are left to colour the graph. Thus, the graph of Fig. 5 needs only three colours and hence this graph is 3-colourable. $L(h_1, h_2, \ldots, h_m)$-labeling is an extension of vertex colouring problem. An $L(h_1, h_2, \ldots, h_m)$-labeling of a graph $G = (V, E)$ is a function f from its vertex set V to the set of non-negative integers such that $|f(x)| - |(y)| \geq h_i$ if $d(x, y) = i$ for $i = 1, 2, \ldots, m$. The span of an $L(h_1, h_2, \ldots, h_m)$-labeling f of G is $\max\{f(v) : v \in V\}$. The $L(h_1, h_2, \ldots, h_m)$-labeling number $\lambda_{h_1, h_2, \ldots, h_m}(G)$ of G is the smallest non-negative integer p such that G has a $L(h_1, h_2, \ldots, h_m)$-labeling of span p.

Vertex Colouring :

Now, we discuss the colouring of some special types of graphs. Path P_n : Let P_n be a path with n vertices $v_1, v_2, \ldots, v_n$. Now, we can colour the vertices with odd indices by an arbitrary colour, say, blue(B) and the vertices with even indices by yellow(Y) as shown in.

Therefore, $c(P_n) = 2$, for $n \geq 2$.

Figure 6 : P_n is 2-colourable, $n \geq 2$.

Cycle C_n :

Any cycle C_n with even number of vertices can be coloured by only two colours like P_n. But, if the number of vertices be odd then exactly three colours are needed to colour C_n. Let the vertices of C_{2k+1} be $v_1, v_2, \ldots, v_{2k}, v_{2k+1}$.

We assign the colour to the vertices $v_1, v_3, \ldots, v_{2k+1}$ by say violet(V) and $v_2, v_4, \ldots, v_{2k}$ by, say yellow(Y). The vertex v_{2k+1} are adjacent to both v_1 (with colour V) and v_{2k} (with colour Y). So, to colour the vertex v_{2k+1} needs one more colour, say B. Thus $c(C_{2k}) = 2$ and $c(C_{2k+1}) = 3$ as shown in.

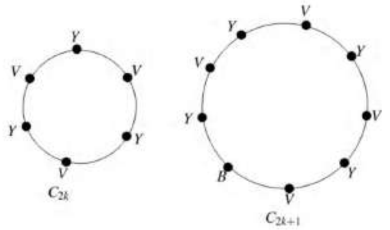

Figure 7 : Colouring of C_n

Complete graph K_n :

Since in complete graph every vertices are adjacent with all other vertices. So, to colour n vertices of K_n needs exactly n colours. Thus, $X(K_n) = n$.

Results and discussion :

Traffic Signal Lights :

Graph colouring is used in traffic signal lights problem. For studying traffic control problems at an arbitrary point of intersection, it has to be modeled mathematically using a simple graph for the traffic accumulation data problem. The rudimentary map edges will represent the communication link between the set of vertices at an intersection point. In the graph, set for the traffic control problem, an edge will join the traffic streams that may move simultaneously at a corner without any difference.

An edge will not connect the streams that cannot move together. The function of traffic lights is turning Green/Red/Yellow lights and timing between them. Here vertex coloring technique is utilized to solve time and space by identifying the chromatic number for the number of cycles needed.

Aircraft Scheduling :

Graph coloring can be used to schedule k aircrafts which has to assign to n flights. The pth flight is at the time interval (a_p, b_p). If two flight overlap then we cannot assign the aircraft. In this case interval graph is used and flights are represented by vertices. The two vertices are connected by an edge, if the time intervals of flights are overlapped. The interval graph can be optimally coloured in polynomial time.

To clear road blockage :

Graph theoretic idea is used to clear road blockage. When roads of any town/city are closed due to snow fall, planning is required so that the roads are clear to put salt on the roads. Then Euler paths or Euler circuits are used to traverse the streets efficiently.

Conclusion :

In this paper, we have discussed a lot of problems that is efficiently solved by applying graph theory and most of them can be solved by using colouring of graphs. There are many applications of graph theory in different branches like chemistry, physics, biology and economics, etc.

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