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Role of Perception in Applications of Labelled Trees

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Abstract:

This paper explores the role of perception i.e., the ways of viewing a particular labelled tree in devising its applications. The different angles of viewing can result in different applications. Three classes of trees namely Arrow, Thorn and Bush labelled with antimagic graph labelling have been considered for this study.

Keywords: Antimagic graph labelling, Arrow Tree, Thorn Tree, Bush Tree

1. Introduction:

Krishnaa [2] gives instances of *antimagic* labelling and the *modified antimagic* labelling situation which highlights the role of *perceptions and the observer* for the series of newly defined Necklace graphs for various scenarios. Section 2 presents the Main Results for the role of different perceptions in the applications of the Arrow, Thorn and Bush Trees discovered in Yadav and Krishnaa [1], while Section 3 presents the Conclusion.

2. Main Results:

Considering the Star graph, the same graph can be drawn as a Complete Bipartite graph $K_{1,5}$ classified in a different category of Bipartite graphs when perceived differently as shown in **Figure 1** and **Figure 2**.

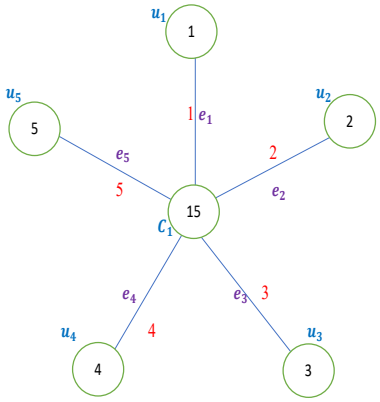


Figure 1: Antimagic Star graph

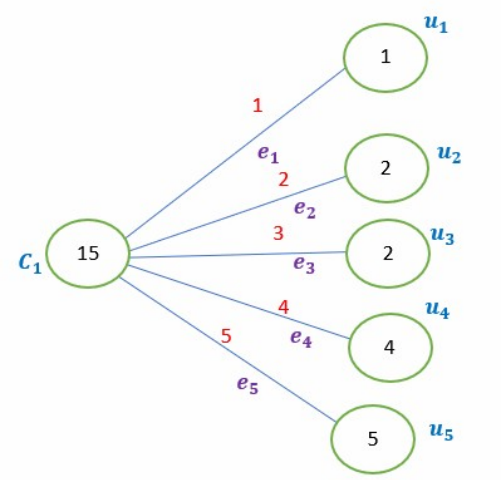


Figure 2: Antimagic Bipartite graph $K_{1,5}$

In this section, applications of classes of Antimagic Trees namely Arrow, Thorn and Bush are presented when these graphs are considered from different viewing angles.

The Horizontal and Vertical views of the Arrow Tree are presented in subsection 2.1, Horizontal and Vertical views of the Thorn Tree in subsection 2.2 and the different perceptions of the Bush Tree in subsection 2.3.

2.1 Definition of Arrow Trees:

Arrow ($A_{m,n,k}$) is a kind of Tree where, m: number of layer(s) on each side of the Central Line termed as Upper Layer(s) and Lower Layer(s). The Upper (Lower) Layer of edges farthest from the Central Line is Upper (Lower) Layer 1, the next closer layer to the Central Line is Upper (Lower) Layer 2, …, Upper (Lower) Layer m namely UL_m(LL_m); n is the number of vertices on each Layer; $k = n + 1$ is the number of vertices on Central Line.

An *antimagic* Arrow Tree is shown in **Figure 3.**

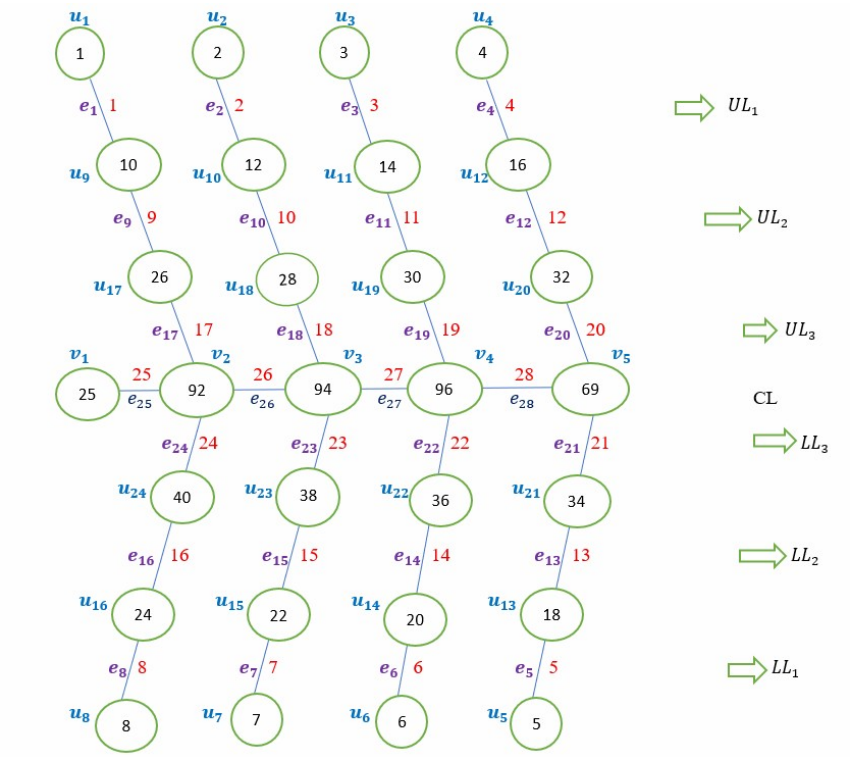


Figure 3: Antimagic ($A_{3,4,5}$) or Tree T(29, 28; 29 vertices, 28 edges) when viewed Horizontally

Buildings in a High Security Area: Vertices are the buildings/offices and edges are the streets in this Arrow Tree.

Horizontal Viewing:

When the Antimagic Arrow Tree ($A_{3,4,5}$) is viewed horizontally as in **Figure 3** then a certain application emerges which is given as follows:

When viewed horizontally i.e., going from Left to Right, can imply increasing secrecy hence increasing security. Vertex labelled 32 is more secure than vertex labelled 30 - this is true for all the Layers above the Central Line. Same holds true for Layers below the Central Line (from Right to Left). The vertex labels can give the identifying location of an office also.

Vertices labelled 10, 12, 14, 16 (above the Central Line) can correspond to the vertices labelled 24, 22, 20, 18 (below the Central Line). Higher number can be taken with "higher security". For example, 20 is more secure than 16, 14, 12.

Vertices labelled 26, 28, 30, 32 (above Central Line) can correspond to the vertices labelled 34, 36, 38, 40 (below the Central Line as in mirror images). Higher number implies "more secure".

Vertical Viewing:

Figure 4 shows the vertical view of the Antimagic Arrow Tree ($A_{3,4,5}$) resulting in different kinds of applications suitable for this angle of viewing.

Office Scenario:

Increasing vertex labels downwards can imply decreasing importance of the personnel hence lesser access to the Head of the Department (vertex labelled 69).

Urban Planning Scenario:

Vertices to the Left of the Central Line can signify "lesser security" than the vertices to the Right of the Central Line. For example, the topmost Left vertices to 69 are labelled with 4, 16, 32 and the corresponding topmost Right vertices are 5, 18, 34 respectively such that $4 < 5$, $16 < 18$, $32 < 34$ signifying the "higher degree of security" to higher numbers on the Right side.

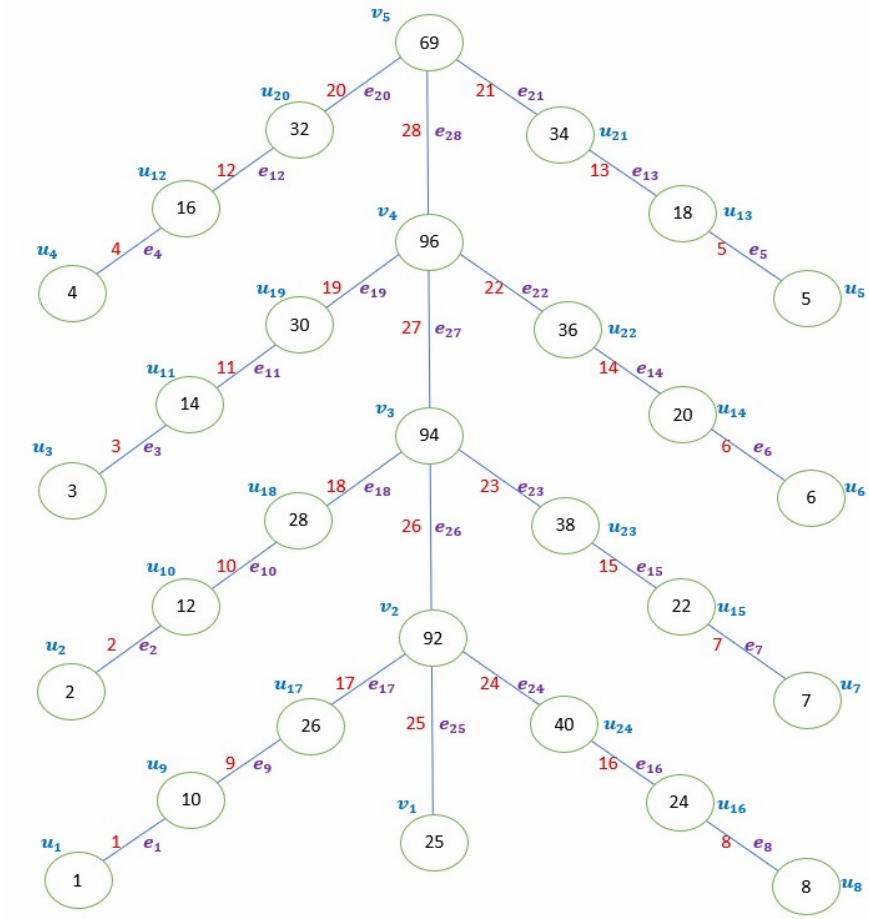


Figure 4: Antimagic ($A_{3,4,5}$) or Tree T(29, 28; 29 vertices, 28 edges) when viewed vertically

2.2 Definition of Thorn Trees:

Thorn ($\boldsymbol{T_{m,n,k}}$) is a kind of Tree where $\boldsymbol{m}$: Total number of vertices on each Layer on both sides of the Central Line termed as Upper Layer(s) and Lower Layer(s). $\boldsymbol{n}$: Number of Layers on each side of the Central Line, The Upper (Lower) Layer of edges farthest from the Central Line is Upper (Lower) Layer 1, the next closer layer to the Central Line is Upper (Lower) Layer 2, …, Upper (Lower) Layer m namely UL_n(LL_n). $\boldsymbol{k}$: Total number of vertices on the Central Line.

An antimagic Thorn Tree is shown in **Figure 5.**

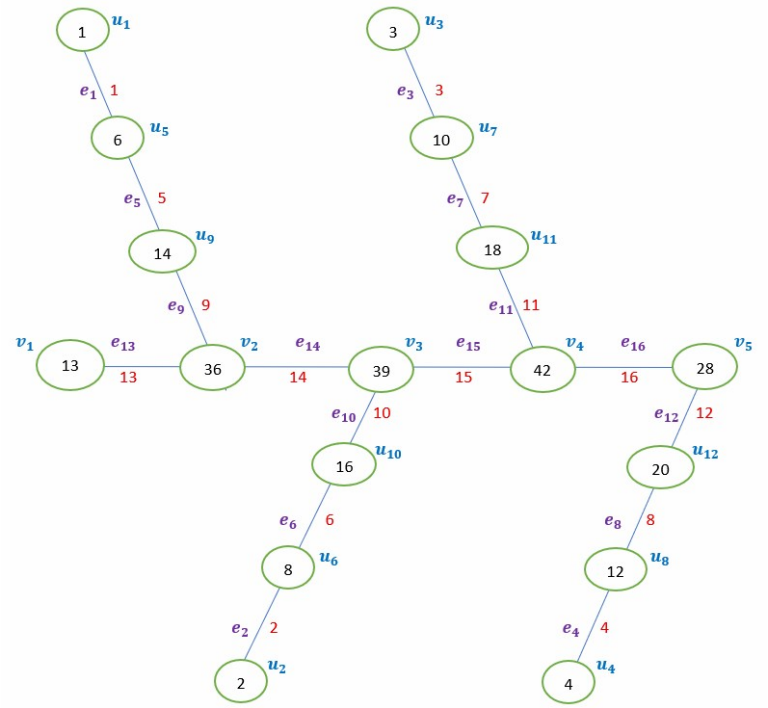


Figure 5: Antimagic Thorn Tree ($T_{4,3,5}$) or Tree (T(17, 16); 17 vertices, 16 edges) when viewed horizontally

The vertical perception of the Antimagic Thorn Tree ($T_{4,3,5}$) different from the horizontal perception (of the same graph) can give rise to a different application (compared to the horizontal perception) which is as follows:

Horizontal Viewing:

Buildings in a High Security Area:

Vertices are the buildings/offices and edges are the streets in this graph. When viewed horizontally i.e., going from Left to Right can imply increasing secrecy hence increasing security.

Vertices labelled 6, 10 in upper layer (above Central Line) can correspond to the vertices labelled 8, 12 in the lower layer (below the Central Line). Similarly, vertices 14, 18 above Central Line can correspond to the vertices 16, 20 (as mirror images).

Vertex label may denote the "level of security". For example, 16 is more secure than 14 and 18 is more secure than 16 and 14.

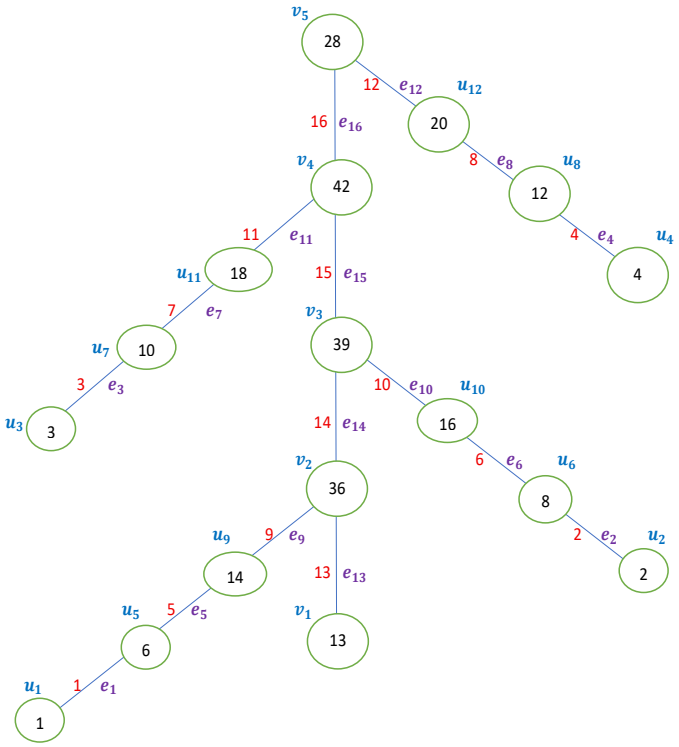


Figure 6: Antimagic Thorn Tree ($T_{4,3,5}$) or Tree (T(17, 16); 17 vertices, 16 edges) when viewed vertically

Vertical Viewing:

Buildings in a High Security Area: Vertices are the buildings/offices and edges are the streets (**Figure 6**).

When viewed vertically i.e., going from Bottom to Top, it can imply increasing secrecy hence increasing security. The Reporting Office is 28 at the top.

Offices closer to the Central Line are more important so have to be more secure. For example, 42 going towards left- 18 is more secure than 10 and 10 is more secure than 3.

2.3 Definition of Bush Tree:

Bush Tree $B(l, m, n)$: From a Central vertex C, l number of edges joined to m no. of edges which in turn are joined to n no. of pendant edges.

A Bush Tree is shown in **Figure 8** and the same Bush Tree as a Rooted Tree is shown in **Figure 7.**

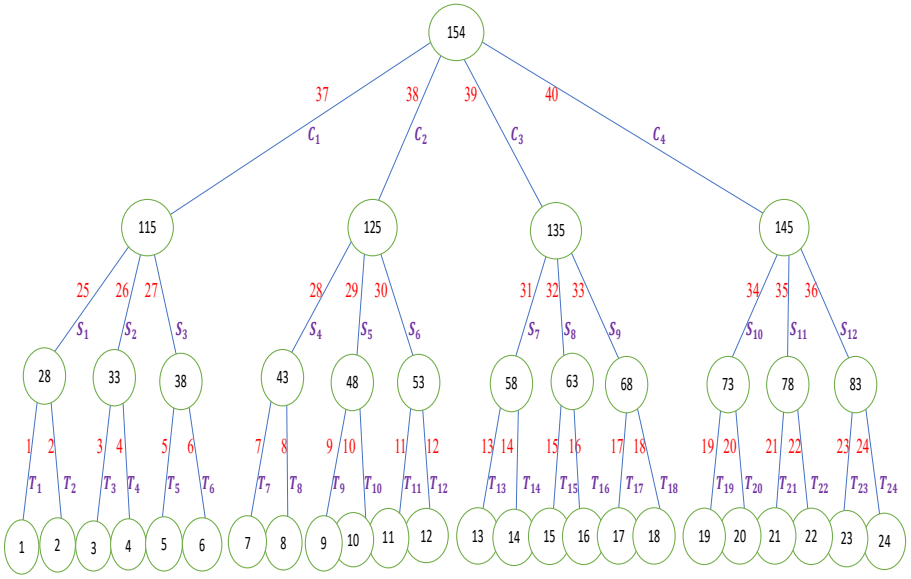


Figure 7: Antimagic Bush Tree $B(4, 3, 2)$ as a Rooted Tree (with hierarchy)

Consider the scenario of a student who has to choose a College, Teacher and Subject.

$C\,i$: i = 1 to 4 (Colleges)

$T\,i$: i = 1 to 24 (Teachers)

$S\,i$: i = 1 to 12 (Subjects)

For a student with high marks, he/she has a choice of College, which has a certain Subject and Teacher. If the student has high marks and is in a position to choose then a certain hierarchy comes into existence and this situation can be modelled as a Rooted Tree shown in **Figure 7**.

For a student with low marks, he/she may not be in a position to choose so this case may be modelled as a Bush Tree proper as shown in **Figure 8**, a graph circular in nature and without hierarchy.

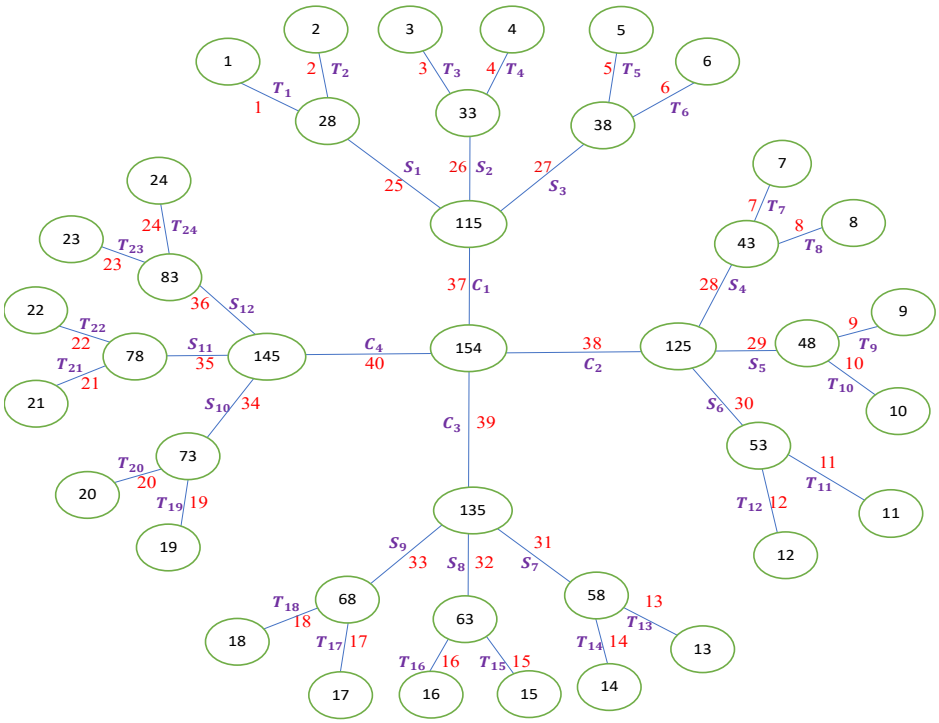


Figure 8: Antimagic Bush Tree *B*(4, 3, 2)

3. Conclusion:

The same graph when perceived differently i.e., viewed from horizontal and vertical viewing can result in different application as in Antimagic Arrow, and Antimagic Thorn Trees. The Antimagic Bush Tree can be drawn as a Rooted Tree and as the originally designed Bush Tree for different applications depending on the presence and absence of hierarchy respectively. Therefore, different perceptions of the same graph can result in different applications.

References:

1. Bharat Kumar Yadav and Auparajita Krishnaa (2024) : Some New Classes of Antimagic Trees, *The Mathematics Education,* Vol. LVIII, No. 3, Sept. 24, pp. 17-47.

2. Auparajita Krishnaa (2025) : Antimagic Labelling and Modified Antimagic Labelling for Necklace Graphs: Patterns, Perceptions and the Observer, *International Journal of Scientific Research in Mathematical and Statistical Sciences*, 12(4), 1-10.