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The Use of Differential Equations to Solutions in Game Theory

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Abstract :

In this paper, the use of differential equation to solution in Game theory. Dockner, E; Jorgenson, S; Vanlong N; Sorger, G. "Differential games in Economics and management science" [1]. An approach of this problem concentrates on the use of discrete transfer schemes to study how players in the game might arrive at a desirable outcome and some interesting theorems.

Keywords : Differential equations and centroids, analytical geometry, system of differential equations, normal vector. Game theory.

1. Introduction :

Differential Equations and Centroids :

Definitions :

Let E^n be Euclidean n-space and $\{a^i\}$ be a fixed set of unit vectors in E^n where

$$i = 1, 2, 3, \ldots\ldots, m.$$

The following functions are defined as :

$$h^i(x, b) = < a^i.x > + b_i$$

Where $x \in E^n$;

$$b = \{b_1, b_2, b_3, \ldots\ldots, b_m\} \in E^m$$

And < . > is the standard inner product on E^n.

We also define the following :

$$P^i(b) = \{x | h^i(x, b) = 0\}$$

And $$Cr(b) = \{x | h^i(x, b) \leq 0\}$$

Clearly, the above $p^i(b)$ is a hyperplane in E^n.

If $Cr(b) \neq \phi$ then it is possibly an unbounded polyhedron in E^n since it is the intersection of half-spaces.

From analytic geometry, the following results are obvious :

(i) The normal (perpendicular) Euclidean distance from any point $x \in E^n$ to $p^i(b)$ is $| h^i(x, b) |$ (absolute value).

(ii) The normal vector from any point $x \in E^n$ to $p^i(b)$ is $- h^i(x, b)\, a^i$

Let $$E_+^m = \{k \in R^m / K_i > 0, i = 1, 2, 3, \ldots\ldots, m\}$$

We consider the following system of differential equations :

$$\frac{dx}{dt} = \dot{x} = B(x, b, k) = -\sum_{i=1}^{m} K_i [h^i(x, b)]^+ a^i$$

Where $[.]^+ = \max\{. , 0\}$.

Clearly the above $B(x, b, k)$ is continuous. Hence the following result follows :

For any $b \in E^m, K \in E_+^m, x^0 \in E^n$, there exists a unique solution $r(t, x_0, b, k)$ to (1), continuous in $t \in (-\infty, \infty)$ and such that

$$r(0, x_0, b, k) = x_0$$

Geometrically, the following half-space $\{x | h(x, b > 0)\}$ can be imagined to be the "wrong side" of hyperplane $p^i(b)$. All other points will constitute the "right side". At any point $x \in E^n$, consider all those i such that x is on the wrong side of $p^i(b)$.

We call such $p^i(b)$ an "offended" hyperplane. We take a positive linear combination of the normal vectors from x to the offended hyperplanes to obtain

$$-\overset{m}{\underset{i=1}{Z}} K_i [h^i(x, b)]^+ a^i$$

Thus the solutions of system $(1a)$ tend to move toward the offended hyperplanes as t increases, ignoring the others. So it might be expected that, along solutions the distance to offended hyperplanes would tend decrease.

2. Centroids Definition :

With the help of above $\{a^i\}$, b and k, we define the set of k-centroids of b with vectors $\{a^i\}$ denoted by $C(b, k)$ as follows :

$$C(b, k) = \{x \in E^n / \phi(x, b, k) = y \in E^n / \phi(y, b, k)\}$$

Where $$\phi(y, b, k) = \sum_{i=1}^{m} K_i([h^i(y, b)]^+)^2.$$

We observe that

(i) If $Cr(b) \neq \phi$, then $Cr(b) = C(b, k)$.

(ii) $C(b, k)$ is independent of k.

In general however, $C(b, k)$ is not independent of k.

Theorem 1 : For any $b \in E^m$ and $k \in E_+^m$, $C(b, k) \neq \phi$.

Proof : One can observe that the problem

$$\inf_{Y \in E^n} (y, b, k) \text{ can be written}$$

$$\inf_{\substack{Z \in E^m \\ Y \in E^n}} \sum_{i=1}^{m} k_i \, Z_i^2$$

subject to $Z_i \geq 0$

$$Z_i \geq h^i(y, b), \quad i = 1, 2, 3, \ldots\ldots, m.$$

The objective function of the rewritten problem is a convex quadratic function, bounded below and the constaints define a nonempty polyhedral convex set.

Since $[.]^+$ is a convex, non-negative and non-decreasing function on R and $(.)^2$ is convex while $h_i(x, b)$ is an affine function of x, it follows that $\phi(x, b, k)$ is also a convex function in x. Also we observe $([.]^+)^2$ is continuously differentiable with

$$\frac{d}{ds}([s]^+)^2 = 2[s]^+.$$

Thus, $\phi(x, b, k)$ is continuously differentiable on E^n.

Let $X = f(x)$ be any system of differential equation on E^n. A^u critical point^n of the system is any point y such that $f(y) = 0$.

Here we see that if ϕ is convex and differentiable continuously then the following holds :

X_n is a k-centroid of b if and only if $\nabla\, \phi(x, b, k)$ such that $x_0 = 0$, where ∇ is the gradient operator with respect to x.

Theorem 2 : x_0 is a k-centroid of b if and only if x_0 is a critical point of system $(1.a)$.

Proof : Since, $\frac{\partial}{\partial x_i}(\delta(x, b, k)) = 2\sum_{i=1}^{m} k_i [< a^i . x > + b_i]^+ a_j^i$

Thus, $\nabla\, \phi(x, b, k) = -2B(x, b, k)$.

So, x_0 is a critical point if and only if $B(x, b, k) = 0$

i.e. iff $\nabla\, \phi(x, b, k)$ such that $x_n = 0$

i.e. iff x_n is a k-centroid of b.

We non-establish some properties of $C(b, k)$. We observe that if $Cr(b) \neq \phi$, then the set of k-centroids of b is a polyhedron. This is true even if $Cr(b) = \phi$.

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