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Discerning the 'Shape' of Mars: Is it 'Pear-shaped', 'Tetrahedroid-shaped' or some other shape?

by **Arjun Tan,** *Department of Physics,*
Alabama A & M University, Normal, AL 35762, U.S.A.
E-mail: arjun.tan@aamu.edu , arjun.tan@mail.com

Ikenna Chris-Okoro and **Ryan Moxon**
Department of Mechanical Engineering,
NCA & T University, Greensboro, NC 27411, U.S.A.

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Abstract:

There exist two descriptions of the 'figure of the Earth' in the literature. First, the notion of the 'Pear-shape of the Earth' is relatively well-known to the public-at-large since the early days of satellite launches. The southern hemisphere bulges out more than the northern hemisphere on average. Further, the north polar area is 18 m higher from the sea-level whereas the south polar area is 28 m below the sea level. When the altitudes are averaged over circles of latitude, the so-called 'pear-shape of the Earth' emerges about the rotational axis of the planet. Second, there exists another description of the figure of the Earth called the 'Tetrahedroid-shape of the Earth', which actually predates the 'pear-shape' concept by 84 years. In 1875, William Lowthian Green proposed the Tetrahedroid-shape of the Earth in order to explain the distribution of land and water on the planet earth. It was hypothesized that a 'law of least action'

would solidify a molten spherical Earth into the smallest 'Platonic solid' or 'Tetrahedroid shape'. This concept received strong historical support before being forgotten. In the tetrahedroid model, one vertex is situated at the north pole, and three others on the circle of latitude 19.5°S, separated by 120°. It should be noted that the tetrahedroid model reduces to the pear-shaped model upon averaging over the longitudes, and is therefore more general than the latter model. In this study, the figure of the planet Mars is investigated. The Martian surface has been thoroughly surveyed by the Martian Orbiter Laser Altimeter (MOLA) aboard the Mars Global Surveyor (MGS) spacecraft for a period of nine years. Topographical maps produced by the MOLA measurements are of the finest quality. A Lambert's equi-angular cylindrical projection map of the Martian surface is used. The coordinate system is defined as a planet centric right-handed spherical polar coordinate system with its z-axis coinciding with the rotational axis of the planet; and the prime meridian passing above the crater Airy-0. The zero elevation is defined as the equipotential surface (formed by gravitational and centrifugal forces) whose average value at the equator is the mean radius of the planet. The results show that the Martian topography was quite similar to that of the Earth, with a major distinction being that its south polar region is an elevated area rather than a depressed one. Consequently, the averaged figure of Mars about its rotational axis is 'lemon-shaped' rather that 'pear-shaped'. Finally, if we ignore the south-polar rise, a 'tetrahedroid-shape' of Mars becomes apparent.

Descriptions of the Shape of the Earth:

The statement that '***the Earth is pear-shaped***' is now fairly well-known to the public-at-large. However, this statement requires some explanation. The shape of the Earth, like that of any other sizeable heavenly body, is determined largely by ***gravitation*** and ***centrifugal force of rotation***. Due to the action of these two forces, the Earth acquires the shape of an ***ellipsoid of revolution*** about its rotational axis, commonly referred to as the ***reference ellipsoid***. Due to other factors such as surface topography, crustal inhomogeneity, land-water distribution, etc., however, the actual surface of the Earth departs slightly, but significantly, from the reference ellipsoid. This was uncovered from the orbital analysis of Vanguard 1 satellite [1]. As for examples, the north pole of the Earth is 18 m above the reference ellipsoid whereas the south pole is 28 m below the latter [1, 2]. When the longitudinal variations are averaged, the latitudinal figure of the Earth attains its pear-shape [1, 2].

Curiously, there is another description of the shape of the Earth, which actually predates the 'pear-shape' concept by 84 years. In 1875, William Lowthian Green proposed the '***tetrahedroid-shape***' of the Earth in order to explain the distribution of land and water on the planet Earth [3]. It hypothesized that a '***law of least action***' would solidify a molten spherical Earth into the smallest '***Platonic solid***' or tetrahedroid-shape. This concept received strong support from Arthur Holmes before being quietly abandoned [3]. Recently, this concept was resurrected by Richard Mentock, who argued that the tetrahedroid-shape of the Earth was more appropriate than the pear-shaped Earth [4]. Arjun Tan determined the best locations of the vertices of the tetrahedron which correspond to ***geoid highs*** in the tetrahedroid model [5]. In this study, we analyze the shape of the planet Mars to determine if it fits the pear-shape, tetrahedroid-shape, or some other shape.

Topography of Mars:

The Martian surface has been thoroughly surveyed by the ***Martian Orbiter Laser Altimeter*** (MOLA) aboard the ***Mars Global Surveyor*** (MGS) spacecraft from September 1997 until November 2006 [6]. Topographical maps produced by the MOLA measurements are of the finest quality. Figure 1 (after [7]) is a ***Lambert's equi-angular cylindrical projection map*** of the Martian surface. The coordinate system is defined as a ***planetocentric right-handed spherical polar coordinate system*** with its ***z-axis*** coinciding with the ***rotational axis of the planet***; and the ***prime meridian*** passing above the crater Airy-0, which serves as the Greenwich-equivalent on Mars (cf., [8]). In Fig. 1, the angular coordinates are: ***zenith angle*** θ $(0 < \theta < \pi)$; and ***azimuth angle*** ϕ $(0 < \phi < 2\pi)$. In the MOLA data, the ***zero elevation*** is defined as the ***equipotential surface*** (formed by ***gravitational and centrifugal forces***) whose average value at the equator is the ***mean radius*** of the planet [9]. In Fig. 1, the surface elevations are marked by color gradations.

The topography of Mars has a smaller degree of complexity than those of the other terrestrial planets. Broadly speaking, the entire surface is divided into two contiguous parts: the lowlands; and the highlands. The lowlands occupy about two-thirds of the northern hemisphere, or one-third of the planet's surface. Most of the lowlands is relatively smooth, deep and sparsely cratered, having an average depth of 4 km [10]. The remaining two-thirds of the planet's surface covering the whole southern hemisphere consists of the highlands. The highlands are rugged with mountains of volcanic origin and highly cratered [10]. For this reason, perhaps, the average elevation of the highlands is hardly mentioned in the literature. But this can be estimated if we assume that the mean elevation of the planet's surface is zero. If $<h>_h$ and $<h>_l$ are average elevations of the highlands and lowlands, respectively; and if A_h and A_l are the surface areas of the highlands and lowlands, respectively, we have:

$$<h>_h A_h + <h>_l A_l = 0 \tag{1}$$

Solving Eq. (1) with $<h>_l = -4$ km, and $A_h = 2\,A_l$, we get: $<h>_h = 2$ km. The highlands are half as high as the lowlands are deep. This information will be used to determine the shape of Mars. An academic exercise of interest will be to determine the circle of latitude which would divide Martian surface into two parts with the northern part consisting of one-third of the planet's surface; and the southern part containing the remaining two thirds. The surface area of the northern part can be calculated as:

$$A_\Theta = \int_0^\Theta r d\theta \int_0^{2\pi} r\sin\theta d\phi = 2\pi r^2(1 - \cos\Theta) \tag{2}$$

where r is the radius of the planet Mars. Setting this equal to one-third of the planet's surface:

$$A_\Theta = \frac{4}{3}\pi r^2 \tag{3}$$

Solving for Θ between Eqs. (2) and (3), one gets: $\Theta = 70.5^\circ$, which gives the required latitude $\lambda = 19.47^\circ$S. This represents the mean latitude of the boundary between the northern lowlands and the southern highlands.

Amongst the notable features on Mars are: ***Olympus Mons***, the highest mountain in the solar system; ***Hellas Planitia***, one of the largest impact craters in the solar system; and ***Valles Marineris***, a 4,000 km long canyon system [11]. A less prominent, but quite significant feature (in relation to this investigation) is the ***north polar plateau*** region northwards of 80°N latitude [12, 13]. This region, centered around the north pole, rises upto 3 km from the surrounding lowlands floor [12] and is made, most likely of, sand, dust and ice, formed perhaps over a billion years [13]. This is significant because it can potentially play the role of the

Earth's north polar region for both the pear-shape and the tetrahedroid-shape models of the Earth's figure. The location of this plateau also fixes the rotational axis of the planet as the figure axis. Figure 2 (after [14]) is a ***Lambert's equal-area projection map*** of the northern hemisphere of Mars. There exists a similar plateau around the south pole, which is just as high as north polar plateau but smaller in area [15].

Method of Investigation:

Having located the figure axes of a potential pear-shape or tetrahedroid-shape and at least one pole or vertex for each, we can now proceed to discern the possible shapes of Mars sought for. The method used to determine the pear-shape of the Moon is used ([16]). Figure 4 is a simplified rendering of Fig.1, where the yellow areas represent the highlands and the blue areas represent the lowlands. The mean elevations over several circles of latitude are estimated in the pear-axis coordinates at regular intervals of 15°.

For each circle of latitude, the angular distances over highlands $\Delta\phi_+$ and lowlands $\Delta\phi_-$ are determined graphically, with the constraint condition:

$$\Delta\phi_+ + \Delta\phi_- = 2\pi \tag{4}$$

If h_+ and h_- are the average elevations of the highlands and lowlands, respectively, then the average elevation over the circle of latitude is:

$$<h>_\theta = \frac{\int_0^{\Delta\phi_+} h_+ d\phi_+ + \int_0^{\Delta\phi_-} h_- d\phi_-}{\int_0^{2\pi} d\phi} \tag{5}$$

Replacing the denominator of Eq. (5) by Eq. (4), one has:

$$<h>_\theta = \frac{\int_0^{\Delta\phi_+} h_+ d\phi_+ + \int_0^{\Delta\phi_-} h_- d\phi_-}{2\pi} \tag{6}$$

As assumed earlier, $h_+ = 2$ km; and $h_- = -4$ km.

Inspecting for Pear-Shape of Mars:

Table I shows the calculated mean values of elevations $<h>_\theta$ for the 13 selected circles of latitude. The resulting shape of Mars about its rotational axis is portrayed in Fig. 4. The black circle represents the zero elevation level. The mean elevations averaged over longitudes are shown in color. The purple line in the northern hemisphere represents elevations below the zero level whereas the red line in the southern hemisphere represents the same above the zero level. Overall, the shape of Mars resembles the pear-shape of the Earth with the major difference at the south polar area where the south polar rise is the direct opposite of the corres-ponding area on the Earth's pea-shape figure, where the Antarctic depression is located. In fact, The Martian figure resembles more like a lemon than a pear. ***Our conclusion is that Mars has a lemon-shaped figure about its rotational axis when averaged over the longitudes***.

Inspecting for Tetrahedroid-Shape of Mars:

The pear-shape of the Earth and the tetrahedroid-shape of the Earth have an important commonality, viz., the north pole serves as the 'stem'-location of the pear and also one vertex of the tetrahedroid [5]. It is the same situation in the case of Mars [4, 5]. With one vertex of the tetrahedroid located at the north pole, the three other vertices must lie on the circle of zenith angle 109.5° or the latitude of 19.5°S [17]. The azimuth angles of the vertices must also be separated by angles of 120° [17]. By visual inspection, the four vertices are easily located and shown in Fig. 5. These are at the locations marked by: **1** ($\theta = 0°$, $\phi = 0°$); **2** ($\theta = 109.5°$, $\phi = 27°$); **3** ($\theta = 109.5°$, $\phi = 147°$); and **4** ($\theta = 109.5°$, $\phi = 267°$) [Fig. 5]. It should be pointed out that the longitude of the north pole can be universally placed any-where on the upper boundary of the figure ($0° \leq \phi \leq 360°$). The locations **2** and **3** both have elevations of over 1 km whereas location **4** is at an elevation of over 4 km. All three locations qualify as the vertices of the tetrahedroid on the base of the tetrahedroid ($\theta = 109.5°$). Admittedly, there are locations on Mars northwards of location **4** which are of higher elevations (Fig. 5). These are mostly mountains of volcanic origin, and are not associated with Green's hypothesis of the initial solidification of Mars [3]. On the other hand, the south polar elevation constitutes a contradiction to Green's hypothesis [3]. On the whole, ***the possibility of tetrahedroid-shape of Mars can be favorably argued***.

Table I: Mean Elevations of Martian Surface for Selected Latitudes

θ, deg	$\Delta\varphi_+$, deg	$\Delta\varphi_-$, deg	$\langle h \rangle_\theta$, km
0	0	360	-1.00
15	0	360	-4.00
30	0	360	-4.00
45	64	296	-2.93
60	132	228	-1.80
75	162	198	-1.30
90	240	120	0.00
105	344	16	1.73
120	328	32	1.47
135	308	52	1.13
150	360	0	2.00
165	360	0	2.00
180	360	0	5.00

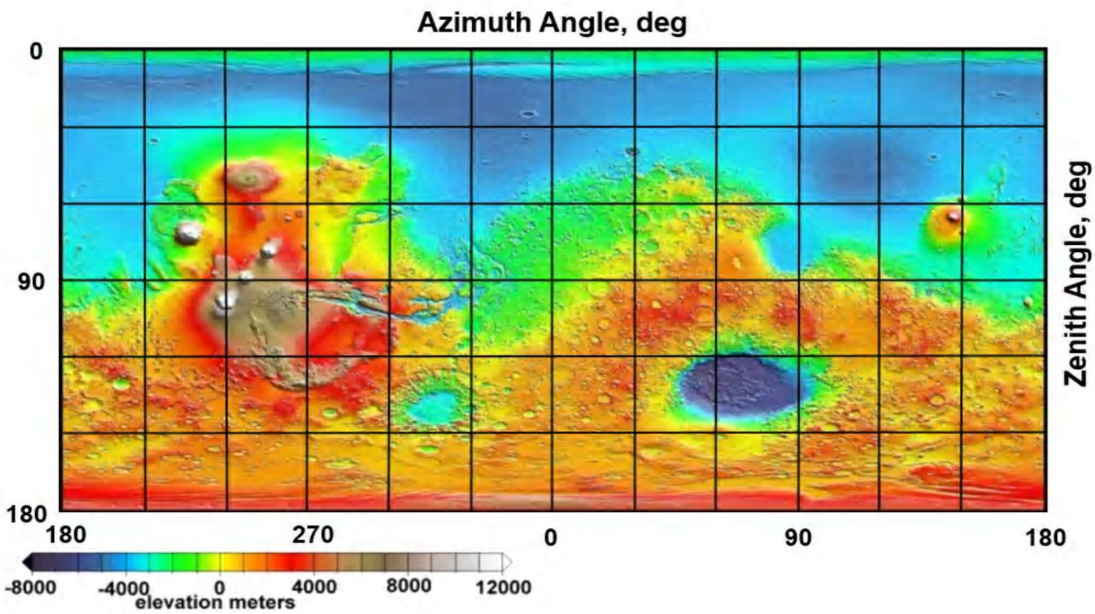

Fig. 1

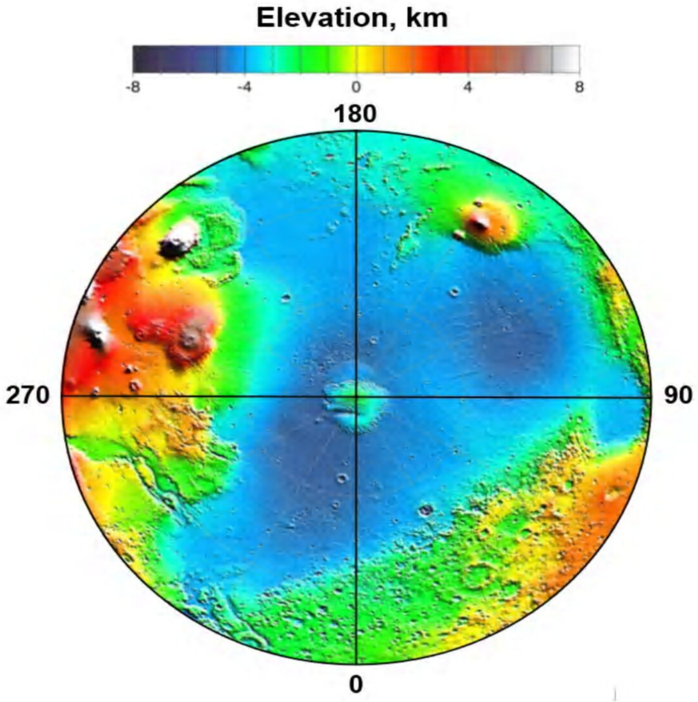
Elevation, km
-8
-4
0
4
8
180
270
90
0

Fig. 2

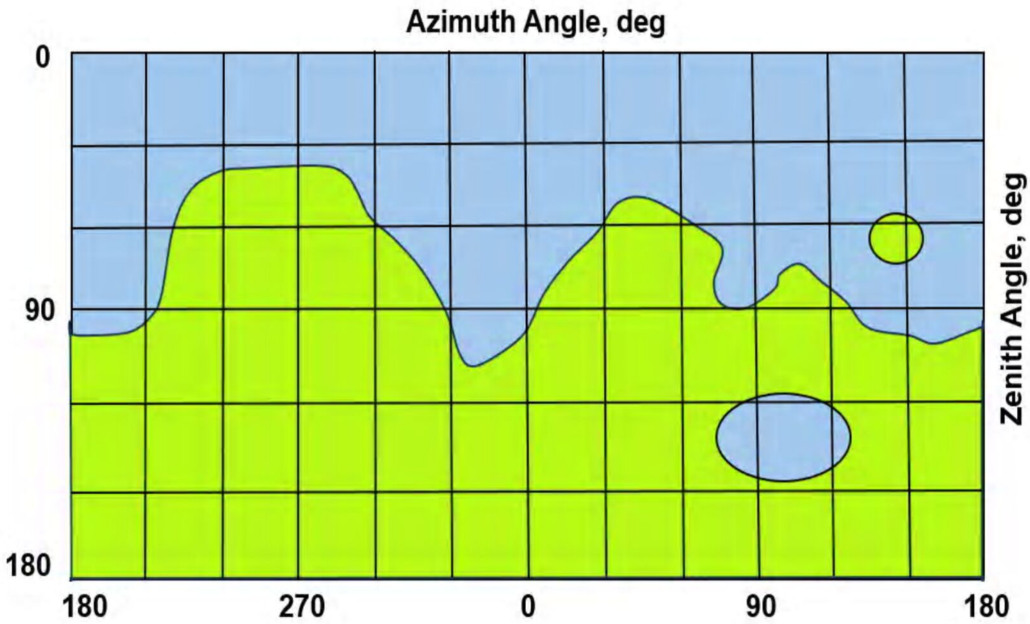
Azimuth Angle, deg
0
90
180
180
270
0
90
180
Zenith Angle, deg

Fig. 3

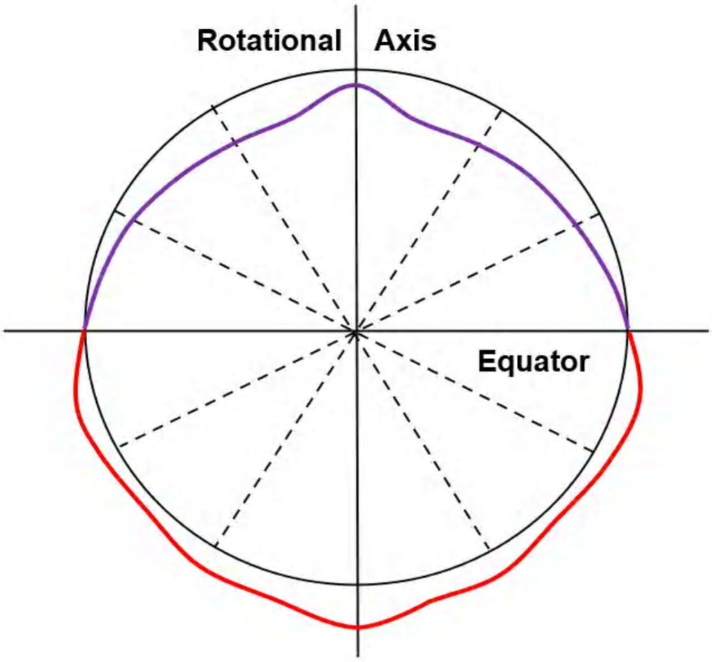
Rotational
Axis
Equator

Fig. 4

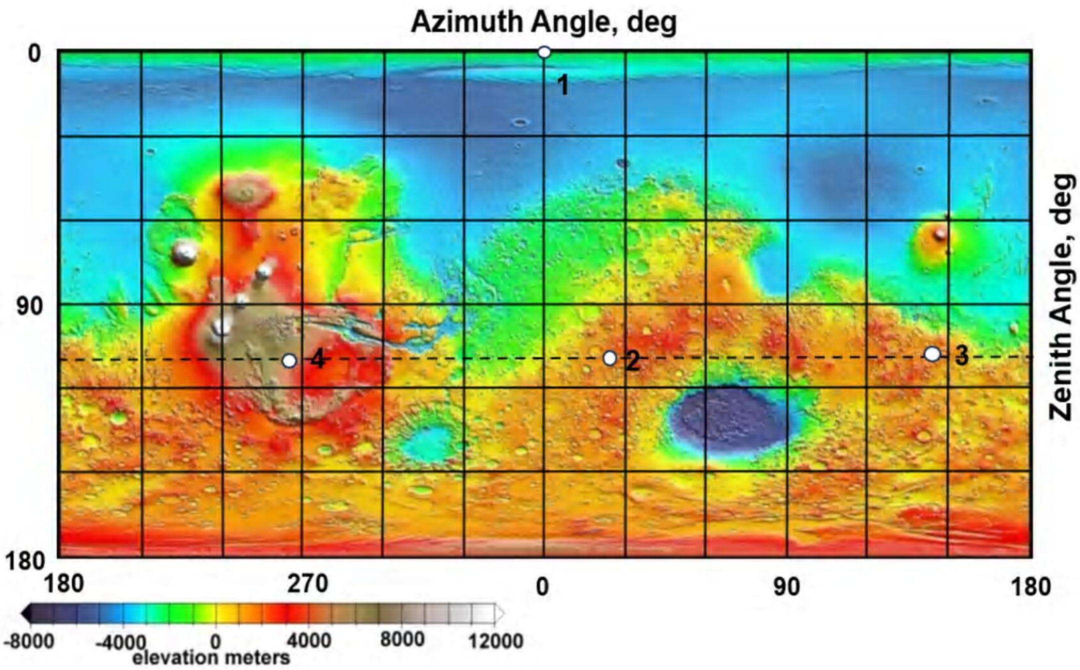
Azimuth Angle, deg
0
90
180
1
2
3
4
180
270
0
90
180
Zenith Angle, deg
-8000
-4000
0
4000
8000
12000
elevation meters

Fig. 5

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