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From the Pear-shape of the Earth to a Tetrahedroid-shape of the Earth

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Abstract:

The notion of the 'Pear-shape of the Earth' is widely known to the public at large. However, a similar concept called the 'Tetrahedroid-shape of the Earth', is not. This concept was recently revived by Mentock, who argued that the tetrahedroid Earth was more appropriate than the pear-shaped Earth. In this study, we construct three model orientations of the tetrahedron within a circumscribed spherical Earth to see which orientation best locates the geoid highs with its four vertices. In the first model (Model A), one vertex is situated at the North Pole, a second on the Greenwich meridional plane and the two others at azimuthal angles of 120° and 240° respectively. Two other models are generated from Model A to better locate the vertices with the geoid highs. The method consists of converting the spherical coordinates of the vertices to rectangular coordinates; then rotating the coordinates system by a desired angle; and finally, converting the new rectangular coordinates back to the spherical coordinates. The origin of the rectangular coordinate system is at the center of *the Earth, with z-axis pointing towards the North Star, the x-axis lying in the Greenwich meridional plane, and y-axis fixed by the right-hand rule. Model B is created by rotating the coordinate system about the y-axis counter-clockwise by –20°; and Model C is created by rotating the coordinate system about the x-axis counter-clockwise by –20°. The locations of the vertices in Models B and C show improved agreements with the geoid highs.*

1. Introduction:

The shape of the Earth, like that of any other sizeable heavenly body, is determined largely by gravitation and centrifugal force of rotation. To the first approximation, owing to the gravitational force alone, the Earth is spherical. To the second approximation, due to the addition of the centrifugal force of rotation, the Earth is an ellipsoid of revolution called the ***reference ellipsoid***. Due to other factors such as land-water distribution, surface topographics, etc., however, the actual surface of the earth departs slightly, but significantly, from the reference ellipsoid. The actual surface is called ***geoid***, which is an equipotential surface. It coincides with the surface of the oceans, and extends under the continents to the level to which ocean water would settle if connected to the ocean by open channels, such as the straights of Gibraltar, Dardanelles and Bosphorus.

Our knowledge about the actual shape of the Earth has been greatly refined by the orbital analysis of the artificial satellites. First, the orbital analysis of Sputnik 2 yielded a more accurate value of the oblateness of the Earth [1]. Next, the orbital analysis of Vanguard 1 uncovered the first indication of the '***pear-shape***' of the Earth, with the stem of the 'pear' located at the North Pole [2].

A sketch of this section is shown in Fig. 1 (adapted from [3]). It should be reminded that there are regional variations of the geoid throughout the globe, and the meridional section of the 'pear-shape' represents the geoid averaged over the longitudes.

Curiously, there already existed another concept of the shape of the Earth, which actually predates the 'pear-shape' concept by 84 years. In 1875, William Lowthian Green proposed the '***Tetrahedroid-shape***' of the Earth in order to explain the distribution of land and water on the planet earth [4]. It hypothesized that a '***law of least action***' would solidify a molten spherical Earth into the smallest '***Platonic solid***' or tetrahedroid shape. This concept strong support from Arthur Holmes before being quietly abandoned [4]. Recently, this concept was revisited by Richard Mentock, who argued that the tetrahedral shape of the Earth was more appropriate than the pear-shaped Earth [5]. In this study, we construct model orientations of the tetrahedron within a circumscribed spherical Earth to see which orientation best locates the geoid highs with its four vertices.

2. Method of Analysis:

In this study, an updated ***geoid map of the Earth*** is used (from [6]). It is drawn on ***Lambert's equi-angular map projection*** [7]. A preliminary examination reveals that there are four major high geoid regions across the globe: one in the northern hemisphere over the north Atlantic Ocean; another over the southern Indian Ocean; a third one over south-eastern Pacific Ocean, extending northwards; and a smaller one over the subduction zone over the Andes mountains. A tetrahedroid shape of the Earth will be verified if it is demonstrated that the four vertices of a tetrahedron inscribed in the Earth are located in the geoid high regions.

In this study, A ***geocentric coordinate system*** is used with the z-axis aligned with the rotational axis of the Earth; the x-axis lying in the Greenwich meridional plane; and the y-axis completing the triad (Fig. 2). The physicists' version of the angular coordinates for any point P on the Earth's surface are used: the ***zenith angle*** $\theta(0^{\circ} - 180^{\circ})$; and ***azimuth angle*** $\varphi(0^{\circ} - 360^{\circ})$. The Tetrahedron is a Platonic solid having four equilateral triangular faces and four vertices [8]. When inscribed within a sphere, the angle subtended by any two vertices at center of the sphere is 109.47° [7]. The pear-shape of the Earth dictates that one vertex of the tetrahedron must lie on the North Pole ($\theta = 0^{\circ}$). In that case, the three remaining vertices must be situated on the circle of latitude ($\theta = 109.47^{\circ}$), separated by 120°. In this study, we construct three model orientations of the tetrahedroid within a circumscribed spherical Earth to see which orientation best locates the geoid highs with its four vertices.

In the first model (named **Model A**), one vertex (V_1) is situated at the North Pole ($\theta = 0^{\circ}$, φ = universal), a second (V_2) on the Greenwich meridional plane ($\theta = 109^{\circ}$, $\varphi = 0^{\circ}$) and the two others (V_3 and V_4) at ($\theta = 109^{\circ}$, $\varphi = 120^{\circ}$) and

($\theta = 109^o$, $\varphi = 240^o$) respectively. Here, the angles have been rounded off to the nearest integers in degrees. The locations of the vertices are marked in Fig. 3. On this map, the vertical axis corresponds to the zenith angle, whereas the horizontal axis corresponds to the azimuth angle. Clearly, all the vertices are located on geoid high regions, but only peripherally so. This is particularly true for the vertices V_1, V_2 and V_3. However, V_4 in the southern Pacific Ocean is located midways between the eastern Pacific Ocean and the Andean mountains highs. Clearly, this calls for better orientation of the vertices on order to justify the tetrahedroid model. This can be achieved by judicious rotation of the coordinate axes as detailed below.

3. Rotation of Coordinate System:

The transformation of angular coordinates (θ and φ) under rotation of the coordinate system consists of three steps as follows (vide Fig. 2):

(1) Find the rectangular coordinates (x, y, z) from the following ***transformation equations*** (setting the radius of the Earth = 1):

$$x = \sin\theta\cos\varphi \tag{1}$$

$$y = \sin\theta\sin\varphi \tag{2}$$

and

$$z = \cos\theta \tag{3}$$

(2) Calculate the rectangular coordinates (x', y', z') in the rotated coordinate system by the following transformation relations:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos(x',x) & \cos(x',y) & \cos(x',z) \\ \cos(y',x) & \cos(y',y) & \cos(y',z) \\ \cos(z',x) & \cos(z',y) & \cos(z',z) \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{4}$$

where the matrix on the R.H.S. is the ***rotation matrix***.

And

(3) Calculate the angular coordinates (θ' and φ') in the rotated coordinate system via the ***inverse transformation relations***:

$$\theta' = \cos^{-1} z' \tag{5}$$

and

$$\varphi' = \tan^{-1} \frac{y'}{x'} \tag{6}$$

4. Results:

A close inspection of Fig. 3 suggests that the vertices V_1 and V_2 of Model A could move southwards to better locations by rotating the coordinate system by -20° about the y-axis. This is done by calculating the angular coordinates (θ' and φ') for the four vertices using Eqs. (1) - (6) to obtain **Model B** giving: V_1' (20°, 0°); V_2' (129°, 0°); V_3' (99°, 124°); and V_4' (99°, 236°). The results display small but not insignificant displacements of V_1', V_2' and V_3' towards the denser parts of the geoid highs (Fig. 4).

We next rotate the coordinate system by -20° about the x-axis and calculate the angular coordinates (θ'' and φ'') to obtain **Model C**. The results for the new vertices are: V_1'' (28°, 317°); V_2'' (127°, 16°); V_3'' (91°, 126°); and V_4'' (115°, 232°). The results show further improvements for V_2'' and V_3'' (Fig. 5).

It should be noted that the angular distance Ψ_{ij} between any two vertices having angular coordinates (θ_i, φ_i) and (θ_j, φ_j), $i \neq j$, remains ***invariant*** under rotation of axes as can be calculated from:

$$\Psi_{ij} = \cos^{-1}[\cos\theta_i \cos\theta_j + \sin\theta_i \sin\theta_j \cos(\varphi_i - \varphi_j)] \tag{7}$$

Any small variation in this study is due to rounding off angles to the nearest degree.

5. Discussion:

(1) The apparently large displacement of V_1 between Models B and C is illusory as the linear distances along the longitudes are elongated on the Lambert's equiangular world map near the Polar Regions.

(2) The concept of a tetrahedroid Earth cannot be summarily dismissed. With one vertex near the North Pole and three in the southern hemisphere, a tetrahedroid Earth, upon averaging longitudinally, can easily reduce to a pear-shaped Earth. The pear-shaped model, on the contrary, cannot yield a tetrahedral Earth.

(3) The so-called 'pear-shape of the Earth' is a reality, produced by the third harmonic in the geopotential expansion of the Earth. It is also strongly correlated to the ***land-water distribution*** of the Earth [9]. Finding a more tangible explanation for the 'tetrahedroid Earth' is perhaps in order.

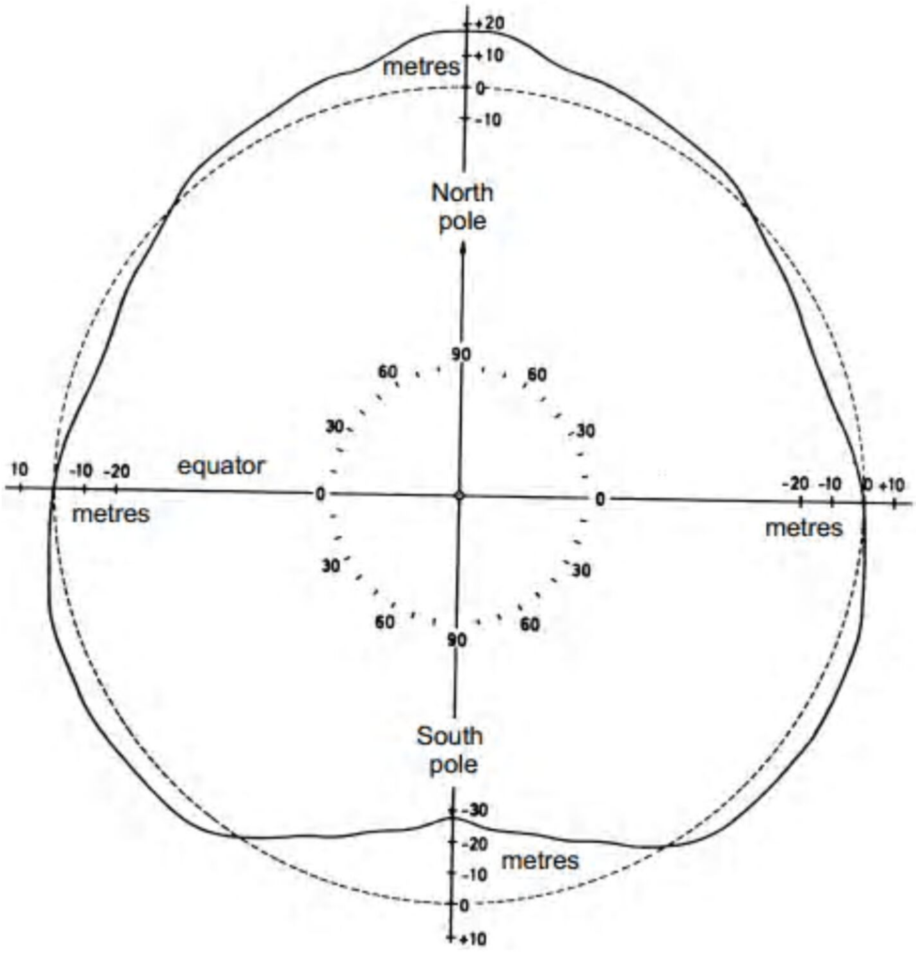

Fig. 1

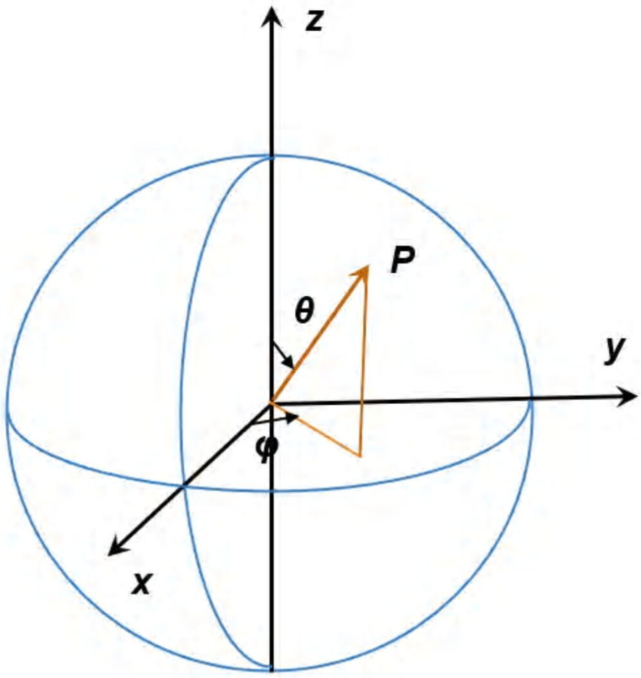

Fig. 2

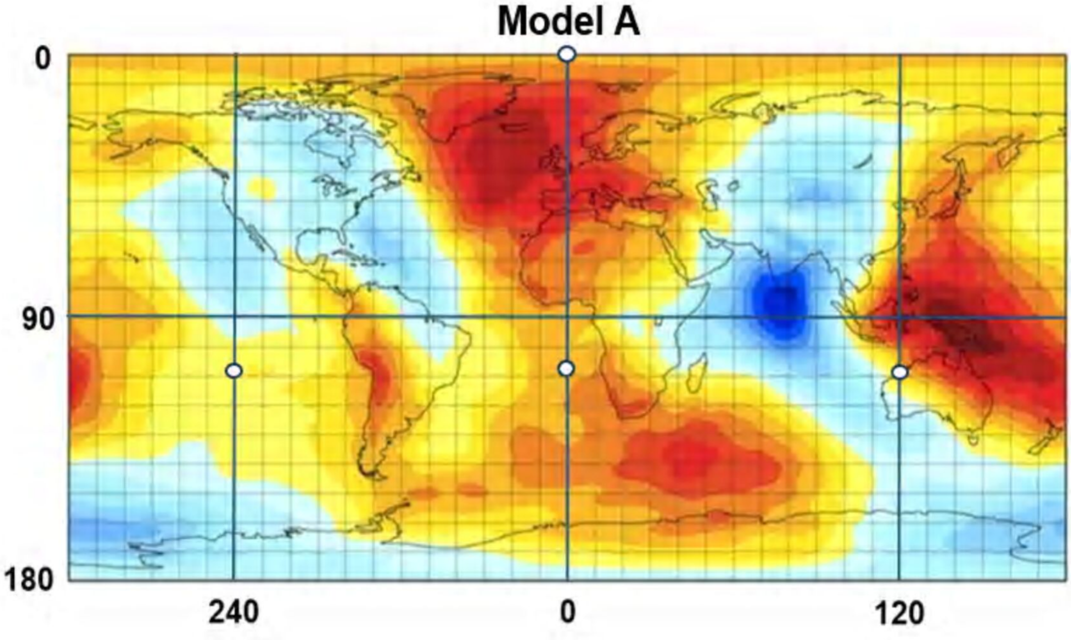

Fig. 3

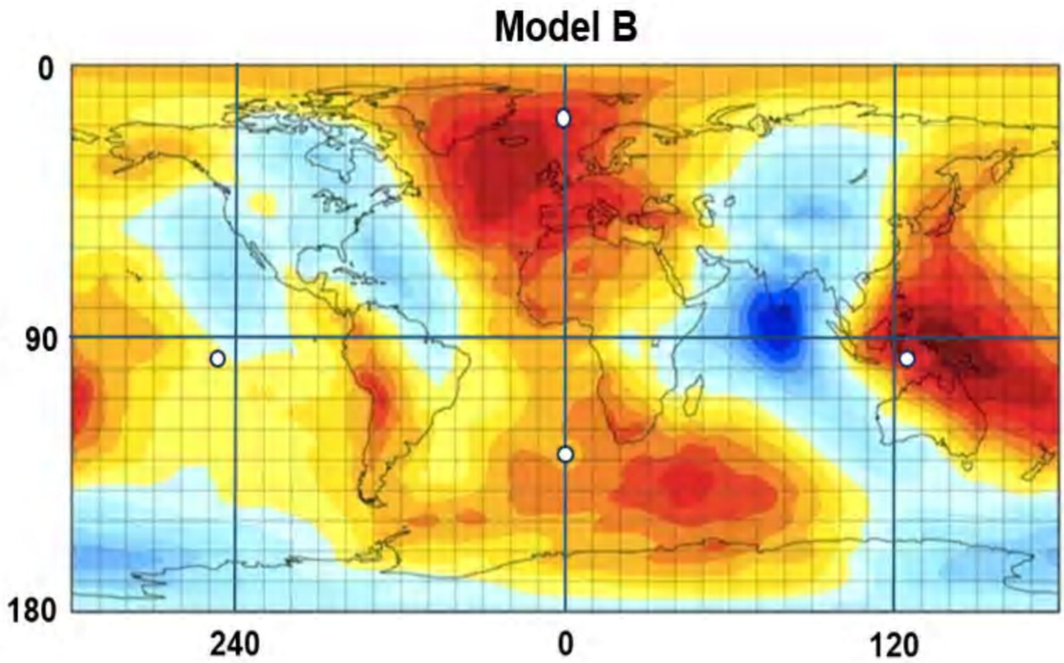

Fig. 4

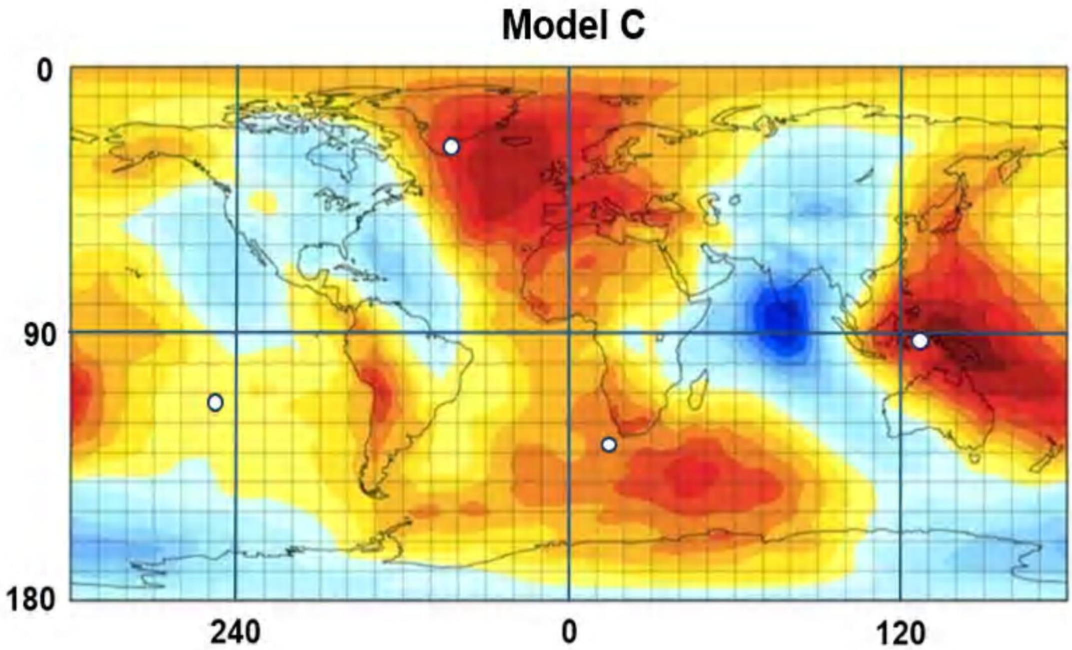

Fig. 5

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