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Unveiling the Pear-shape of the Moon

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Abstract:

The notion of the 'Pear-shape of the Earth' is well-known to many in the field of science. In this study, the possibility of the existence of an analogous 'Pear-shape of the Moon' is investigated. We begin with a topographic map of the lunar surface in Lambert's azimuthal equal-area projection for both the near and far faces of the Moon in selenographic coordinates. A preliminary inspection of this map indicates that a pear-shape of the Moon similar that of the Earth may be possible if the axis of the pear is situated in the prime meridional plane with its north pole tilted 15° towards the direction of the Earth. A Lambert's azimuthal equi-angular map is constructed on which a simplified highland and lowland areas are drawn. Next, the angular coordinates in pear-axis frame of reference are calculated. A simplified elevation model of the lunar surface is assumed, in which the highland area elevations are assumed to be 1 km above the mean elevation and the lowland areas assumed to be 1 km below the mean elevation. The mean elevations of circles of latitude in the pear-axis coordinates are estimated graphically for intervals of 15°. Finally, these mean elevations are displayed around an axisymmetric Moon. An emphatic pear-shape of the Moon strikingly similar to that of the Earth clearly emerges. The most fascinating feature of this pear-shape is that its amplitude is many times greater than that of the terrestrial counterpart.

Introduction:

The shape of the Earth, like that of any other sizeable heavenly body, is determined largely by gravitation and centrifugal force of rotation. To the first approximation, owing to the gravitational force alone, the Earth is spherical. To the second approximation, due to the addition of the centrifugal force of rotation, the Earth is an ellipsoid of revolution called the ***reference ellipsoid***. Due to other factors such as land-water distribution, surface topography, etc., however, the actual surface of the Earth departs slightly, but significantly, from the reference ellipsoid. The actual surface is called ***geoid***, which is an equipotential surface. Our knowledge about the actual shape of the Earth has been greatly refined by the orbital analysis of the artificial satellites. First, the orbital analysis of Sputnik 2 yielded a more accurate value of the oblateness of the Earth [1]. Next, the orbital analysis of Vanguard 1 uncovered the first indication of the '***pear-shape***' of the Earth, with the stem of the 'pear' located at the north pole [2]. A sketch of this section shown in Fig. 1 (from [3, 4]) indicates that the north pole is 18 m above the reference ellipsoid whereas the south pole is 28 m below the latter. In this study, we investigate the existence of a pear-shape of the Moon, if any.

Topography of the Lunar Surface:

The surface of the Moon has been well-studied from the many lunar orbiter missions of the past. Topographic maps of the lunar surface are readily found on the web literature (e.g., [5], [6]). They are usually for the ***near-face*** and the ***far-face*** of the Moon in ***Selenographic coordinates*** [7]. In such a system, the ***orthogonal rectangular coordinates*** are: (1) the ***x-axis*** from the center of the Moon pointing towards the Earth; (2) the ***z-axis*** pointing to the north; and (3) the ***y-axis*** fixed by the ***right-hand rule*** (Fig. 2). The two angular coordinates are: (1) the ***zenith angle*** Θ measured from the north pole ($0^\circ < \Theta < 90^\circ$); and (2) the ***azimuth angle*** Φ measured from the ***prime meridian*** eastwards ($0^\circ < \Phi < 180^\circ$). The latter is, of course, the ***meridional plane*** containing the *x*-axis (Fig. 2).

Figure 3 (adapted from [6]) is a topographic map of the Moon in ***Lambert's azimuthal equal-area projection*** obtained from the ***Clementine mission*** [8]. It is evident that the lunar surface consists of two types: (1) the ***highlands*** (marked by the yellow-orange-red end of the ***visible spectrum***); and (2) ***mares/lowlands*** (marked by the violet-indigo-blue end of the spectrum). The near side consists mainly of lowlands with some highlands around the north pole and in the extreme southern hemisphere. The far side, on the other hand, comprises mostly highlands, but includes a gigantic basin called the ***South-polar-Aitken-basin*** (SPAB), considered the largest impact crater in the solar system [9]. Estimations of the average elevation of the highlands and the average depths of the lowlands are hard to find in the literature. However, it is estimated that the average elevation of the far side is 1.9 km higher than that of the near side [10]. It is further estimated that the average elevation of the highlands is about 3 km higher than that of the lowlands [11]. From these estimates, it will be fair to assume that the average elevation of the highlands and the average depths of the lowlands from the mean elevation of the lunar surface are both about 1 km. We shall use this assumption in this study.

Pear-Shape of the Moon?

A preliminary inspection of Fig. 3 indicates that a pear-shape of the Moon similar that of the Earth may be possible if the stem of the pear can be located at its north pole with the botton of the pear at the south pole. But first some differences should be pointed out. In the case of the Earth, the pear-shape is reckoned relative to the reference ellipsoid of the Earth. In the case of the Moon, the concept of an ellipsoid is non-existent given the fact that its rotation is frozen with the Earth. Next, due to the absence of oceans, the question of land-water distribution is irrelevant. However, these factors actually make the concept of the pear-shape on the Moon simpler. The latter can simply be defined relative to the average elevation of the lunar surface.

A closer examination of Fig. 3 shows that SPAB touches the south pole of the Moon, but does not enclose it completely. For a pear-shape of the Moon similar to that of the Earth to realize, the axis of the prospective pear must be moved closer to its center. However, this move should be limited such that the stem of the pear remains within the highlands of the north polar region and not fall inside the lowlands to the south. A compromise action satisfying both the constraints will be to rotate the axis of the Moon 15° in the meridional plane so that the north pole tilts towards the Earth to the point ($\Theta_0 = 15^{\circ}$, $\Phi_0 = 0^{\circ}$) [vide Fig. 3]. In this rotated frame of reference, called the ***pear-axis coordinate system***, the angular coordinates (zenith angle θ, azimuth angle ϕ) will be different and evaluated in the following.

Pear-Axis Angular Coordinates:

The angular coordinate transformations from the selenographic coordinates (Θ, Φ) to the pear-axis coordinates (θ, ϕ) can be derived via ***spherical trigonometry***. From Ref. [12]:

$$\cos\theta = \cos\Theta\cos\Theta_0 + \sin\Theta\sin\Theta_0\cos(\Phi - \Phi_0) \tag{1}$$

and

$$\sin\phi = \sin\Theta\sin(\Phi - \Phi_0)\text{cosec}\,\theta \tag{2}$$

Since $\Phi_0 = 0$, Eqs. (1) and (2) reduce to:

$$\theta = \cos^{-1}[\cos\Theta\cos\Theta_0 + \sin\Theta\sin\Theta_0\cos\Phi] \tag{3}$$

and

$$\phi = \sin^{-1}[\sin\Theta\sin\Phi\,\text{cosec}\,\theta] \tag{4}$$

Figure 4 is a simplified rendering of the lunar topography in ***Lambert's azimuthal equi-angular projection map***. The yellow areas represent the highlands whereas the blue areas represent the lowlands. The blue straight lines mark the circles of longitude and semi-circles of latitude in the Selenographic coordinates. The north pole of the proposed pear-axis is found near the top of the near side and the south pole is likewise found near the bottom of the far side. The circles of latitude and longitude in the pear-axis coordinates are determined from the calculated values of θ and ϕ from Eqs. (3) and (4) and appear as red lines in Fig. 4.

Construction of the Pear-shape of the Moon:

In order to investigate the existence of the pear-shape of the Moon, the mean elevations over several circles of latitude are estimated in the pear-axis coordinates at regular intervals of 15°. The mean elevation over a circle of latitude θ is obtained by integrating the elevation h_ϕ and taking its average:

$$\langle h \rangle_\theta = \frac{\int_0^{2\pi} h_\phi d\phi}{\int_0^{2\pi} d\phi} = \frac{\int_0^{2\pi} h_\phi d\phi}{2\pi} \tag{5}$$

As stated earlier, for the sake of convenience, we assume that $h_\phi = \pm c$, where the + sign is for the highlands; the – sign is for the lowlands; and $c = 1$ km = 1000 m. Integrating separately over the highlands and lowlands, we have:

$$\int_0^{2\pi} h_\phi d\phi = \int h\phi_+ d\phi_+ + \int h\phi_- d\phi_- = c[\Delta\phi_+ - \Delta\phi_-] \tag{6}$$

where

$$\Delta\phi_+ + \Delta\phi_- = 2\pi \tag{7}$$

whence, from Eq. (5)

$$< h >_\theta = \frac{c}{2\pi}[\Delta\phi_+ - \Delta\phi_-] \tag{8}$$

The average elevations over pear-axis circles of latitude were evaluated this way by graphically estimating $\Delta\phi_+$ and $\Delta\phi_-$ from Fig. 4 for intervals of 15° and entered in Table I.

Table I: Mean Elevations of the Lunar Surface for Selected Latitudes in Pear-Axis Coordinates

θ, deg	$\Delta\varphi_+$, deg	$\Delta\varphi_-$, deg	$\Delta\varphi_+ - \Delta\varphi_-$, deg	$<h>_\theta$, m
0	360	0	360	1,000
15	210	150	60	167
30	110	250	-140	-386
45	125	235	-110	-306
60	170	190	-20	-56
75	220	140	80	220
90	225	135	90	250
105	235	125	110	306
120	250	110	140	389
135	300	60	240	167
150	255	105	150	417
165	90	270	-180	-501
180	0	360	-360	-1,000

Results and Discussion:

The mean elevations $< h >_\theta$ thus obtained for the different circles of pear-axis latitude from Table I are now displayed around an axisymmetric Moon in Fig. 5. An emphatic pear-shape of the Moon strikingly similar to that of the Earth (Fig. 1) clearly emerges! What is most fascinating is that the amplitude of the pear-shape is far greater than that of the Earth. The top of the pear is 1 km above the mean elevation and the bottom of the pear is 1 km below the mean elevation. This is, of course, the consequence of setting the value of the $c = 1$ km. Comparing these values with those of the Earth's pear shape (18 m and 28 m, respectively [3]), one can see that the amplitude of the pear-shape of the Moon is many times greater than that of the Earth. Actually, this difference is even more extreme given the fact that the deepest point of the Moon's surface is actually 9 km below the mean elevation at a location of ($\theta = 160^\circ$, $\phi = 188^\circ$) not far from the bottom of the pear-axis [9].

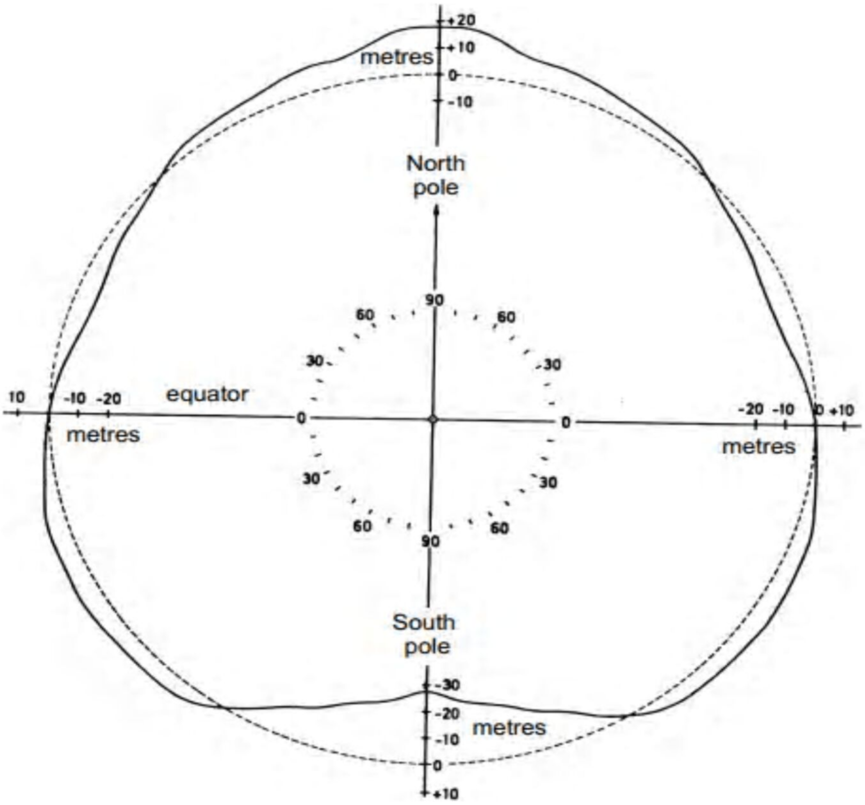

Fig. 1

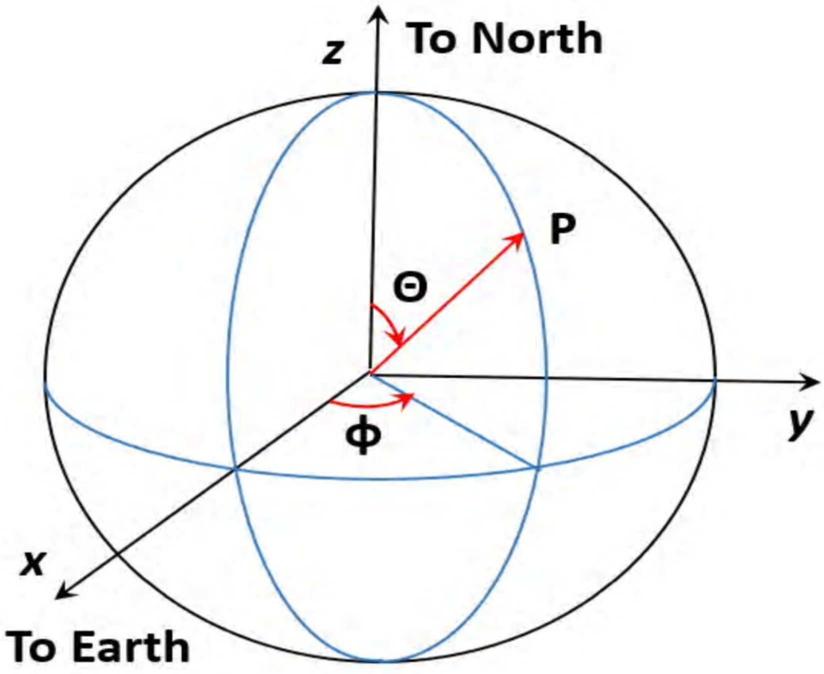
z
To North
P
Θ
Φ
y
x
To Earth

Fig. 2

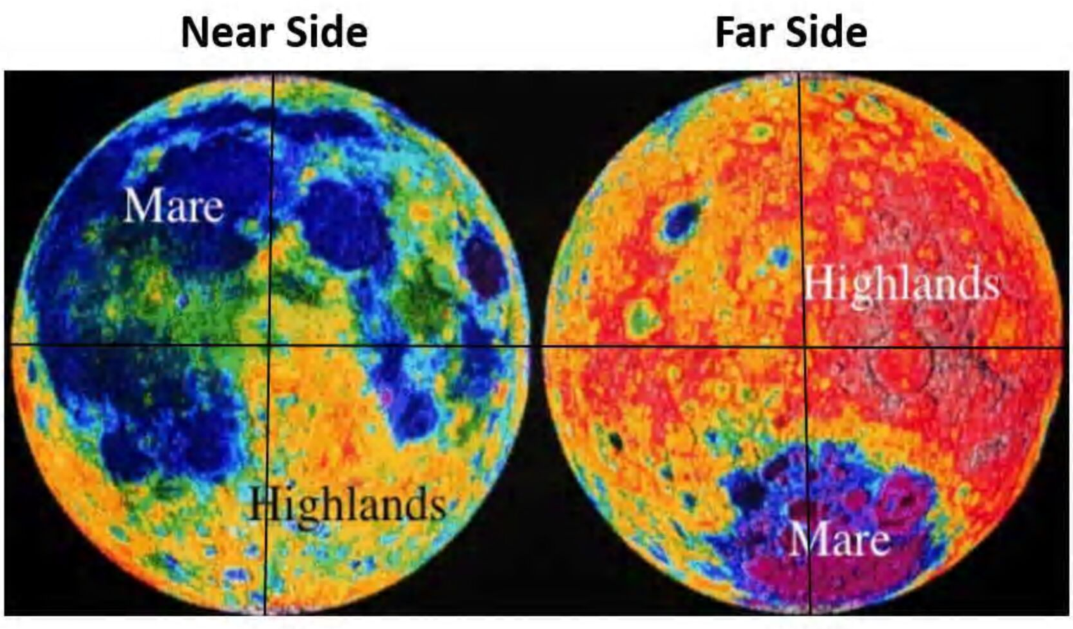
Near Side
Far Side
Mare
Highlands
Highlands
Mare

Fig. 3

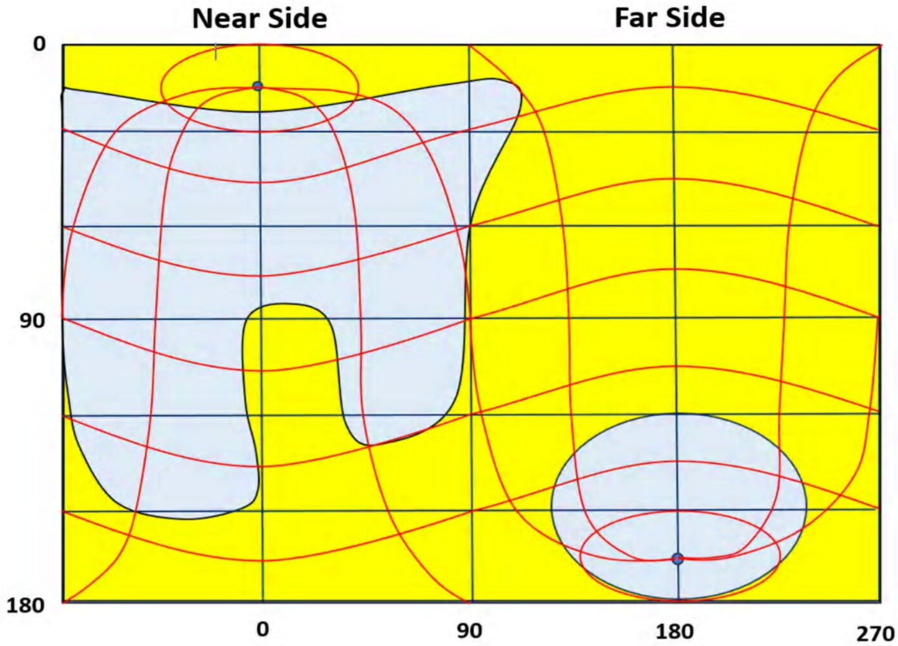
Near Side
Far Side
0
90
180
0
90
180
270

Fig. 4

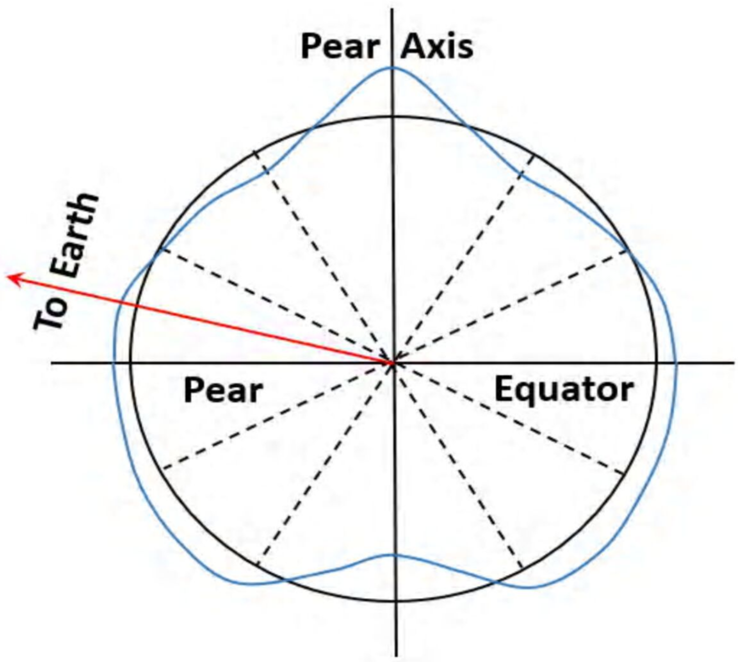
Pear Axis
To Earth
Pear
Equator

Fig. 5

References:

[1] R.H. Merson & D.G. King-Hele : Use of artificial satellites to explore the Earth's gravitational field: Results from Sputnik 2, *Nature*, **182**, 640-641 (1958).

[2] J.A. O'Keegfe, A. Eckels & R.K. Squires : Vanguard measurements give pear-shaped component of Earth's figure, *Science*, **129**, 565-566 (1959).

[3] D.G. King-Hele : Satellite orbits in an atmosphere: Theory and Applications (*Blackie*, 1987).

[4] A. Tan, A. Alomari & M. Schamschula : Land-water distribution and the pear-shape of the Earth, *Int. J. Oceans Oceanogr.*, **10**, 123-132 (2016).

[5] http://commons.wikimedia.org/wiki/File:M00nTopoGeoidUSGS.jpg

[6] https://simonhanmer52.Ca/moon-i--highlands--lowlands.html

[7] https://en.wikipedia.org/wiki/Selenographic_coordinate_system

[8] https://en.wikipedia.org/wiki/Clementine_(spacecraft)

[9] https://usgs.gov/news/feature-story/visit-moon-without-leaving-your-desk

[10] https://en.wikipedia.org/wiki/Selenography

[11] https://www.lpi.usra.edu>BelfryHighSchool

[12] S.I. Akasofu & S. Chapman : Solar Terrestrial Physics (*Oxford, London*, 1972).