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Estimating Solar Zenith Angle and Local Time From a Partial Rainbow

by **Arjun Tan,** *Department of Physics, Alabama A & M University, Normal, AL 35762, U.S.A.*
E-mail: arjun.tan@aamu.edu ; arjun.tan@mail.com

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Abstract:

A method of calculating the solar zenith angle and the local time from the photograph of a full rainbow is found in the literature. This paper describes a method of doing the same from the photograph of a partial rainbow as long as one end of the rainbow meets the horizon. The only measurement needed is the angle between the rainbow end and the horizon from the photograph. The solar zenith angle can be calculated from this angle and the half angle of the rainbow cone. Next, the hour angle of the Sun is calculated given the latitude of the location and the declination of the Sun, whence the local time of the occasion is determined.

Introduction:

Rainbows have fascinated man from time immemorial. They have occupied the minds of scientists and poets alike and romanticized the common public in general. The formation of rainbows can be found in college textbooks in physical science all over the world. Some decades ago, the author of this paper published a method of calculating the solar zenith angle and local time from the photograph of a full rainbow [1]. This paper describes a method of doing the same from the photograph of a partial rainbow, as long as one end of the rainbow meets the horizon. The only measurement required is the angle between the rainbow end and the horizon from the photograph. The solar zenith angle can be calculated from this angle and the half angle of the rainbow cone. Next, the hour angle of the Sun is calculated given the latitude of the location and the declination of the Sun, whence the local time of the occasion is determined.

Method:

In this paper, only the ***primary rainbow*** is considered since it is the brightest bow by far. The ***secondary bow*** is not only much weaker, it is seldom fully formed. Figure 1 gives a geometrical perspective of the primary bow formation. ***O*** is the observation point; ***ON*** is the local vertical; ***OS*** points towards the Sun; and ***A*** is the sub-solar point, at an angle β below the horizontal plane. ***OCF*** is the ***average rainbow cone*** represented by green light having a semi-angle of $\alpha = 41.34^o$ [1]. ***DCEF*** is the rainbow circle; and ***DBE*** is the horizon. Only the part of the rainbow above the horizon ***DCE*** is visible from ***O***. For the partial bow under consideration in this paper, assume that one end of the partial bow meets the horizon at ***D***. Lastly, ***DT*** is the tangent to the bow at ***D*** making an angle $<TDB = \theta$, which is measured from a photograph of the bow by a ***protractor***. The green lines are in the rainbow plane whilst the blue lines are in the local vertical plane.

If β is the ***declination of the sub-solar point*** and ζ is the ***solar zenith angle***, then (vide Fig. 1):

$$\zeta = 90^o - \beta \tag{1}$$

From elementary trigonometry,

$$\tan\alpha = \frac{AC}{OA} \tag{2}$$

and

$$\tan\beta = \frac{AB}{OA} \tag{3}$$

Dividing Eq. (3) by Eq. (2), one gets:

$$\frac{\tan\beta}{\tan\alpha} = \frac{AB}{AC} = \frac{r\text{-}h}{r} = 1 - \frac{r}{h} \tag{4}$$

where r = ***radius of the bow***; and h = ***height of the bow*** (if fully formed).

Next, in the plane of the bow, $<TDA = 90^o$; therefore, $<ADB = 90^o - \theta = <OAD$; and $<DAB = \theta$(vide Fig. 1). Thus,

$$\cos\theta = \frac{r\text{-}h}{r} = 1 - \frac{r}{h} \tag{5}$$

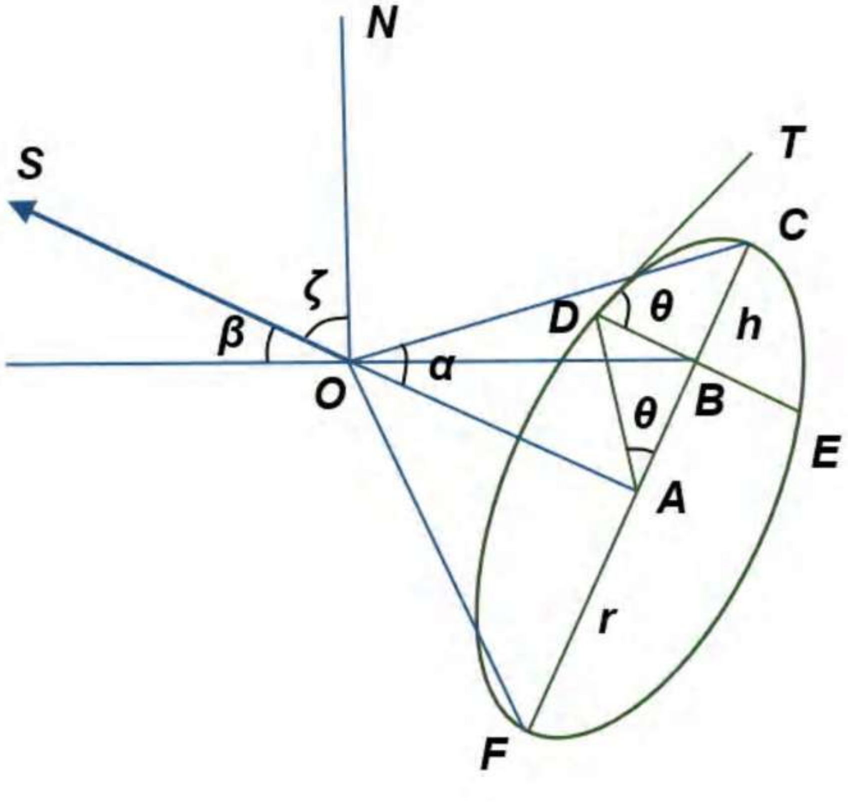

Fig. 1: Schematic diagram of Rainbow formation

From Eqs. (4) and (5), we get:

$$\tan\beta = \cos\theta \tan\alpha \tag{6}$$

Equation (6) furnishes the formula for calculating β, whence the solar zenith angle ζ is obtained from Eq. (1).

The solar zenith angle depends upon the ***latitude of the place*** λ, ***declination of the Sun*** δ and the ***hour angle of the Sun*** A through the equation (see, for example [2]):

$$\cos\zeta = \sin\delta \sin\lambda + \cos\delta \cos\lambda \cos A \tag{7}$$

The hour angle is measured from high noon and is equivalent to 15° for each hour. For example, $A = 45^{\circ}$ corresponds to 3 pm, while $A = -60^{\circ}$ corresponds to 8 am, and so on. Having found the solar zenith angle ζ, the hour angle can be calculated from Eq. (7) given the latitude of the place λ and the declination of the Sun δ on that day:

$$A = \cos^{-1}\frac{\cos\zeta - \sin\delta \sin\lambda}{\cos\delta \cos\lambda} \tag{8}$$

The resulting local time is then determined from the hour angle A.

An Example:

We now illustrate our method of determining the solar zenith angle and local time from a partial rainbow outlined above. Figure 2 is a picture of a rainbow which appeared above Maraetai Beach near Auckland, New Zealand on 20 July 2010 after a downpour (courtesy of Tom & Haley Sulcer [3]). The horizon over the Pacific Ocean is clearly visible. The angle between the end of the rainbow and the horizon θ measured by a protractoris found to be 84°. The coordinates of Maraetai Beach are 36.881°S & 175.042°E [4]. (Here we only need the latitude $\lambda = -36.881^{\circ}$ for our purpose.) Finally, the declination of the Sun on that day δ was 20.83° (from[5]). Calculations carried out using Eqs. (6), (1) and (8) yield: $\beta = 5.25^{\circ}$; $\zeta = 84.75^{\circ}$; and $A = 4.3949$ hr, which translates to the desired local time of 4:23:41.75 pm.

Fig. 2: Rainbow photographed at Maraetai Beach on 20 July 2010

It should be reminded that a solar zenith angle of 90° signifies the moment of sunset. The time of sunset can be estimated from Eq. (8) by setting $\zeta = 90^{\circ}$, whence $A = 73.4135$ hr. This corresponds to a sunset time of 4:53:39.23 pm for Maraetai Beach on 20 July 2010, which was just under 30 min away from the rainbow time. This is not surprising since it was in the middle of winter in the southern hemisphere on that day.

The author wishes to thank Tom and Haley Sulcer for uploading the picture on the web and making it available for free downloading by others.

References:

[1] A. Tan and Tom King: Solar zenith and local time from a Rainbow, *Phys. Teacher*, **28**, 224-225 (1990).

[2] https://thesolarlabs.com/ros/solar-angles/

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[4] https://en.wikipedia.org/wiki/Maraetai

[5] http://solarsena.atspace.com/sundec.htm