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# A Study of (4, 2)-jection Operator

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**Abstract :**

*In this paper, we use notion of projection operator on a linear space as the Simmons G. F. "Introduction to topology and modern analysis". In analogues to the above notions we have introduced a new type of operator called (4, 2)-jection on a linear space. It is a generalisation of a projection operator in the sense that every projection is a (4, 2)-jection, but a (4, 2)-jection is not necessarily a projection. We shall study (4, 2)-jection on $R^2$ and consider different examples of (4, 2)-jection.*

**Keywords :** Linear space, Projection, trijection, tetrajection, and (4, 2)-jection.

## 1. Introduction :

A linear transformation on $L$ into itself, we refer to it as a linear transformation on $L$ . Also $E$ is called the projection on $M$ along $N$, where $N$ is linear subspace of $M$.

Clearly $E^2 = E$.

In analogue to this definition, a trijection operator $E$ has been defined by Dr. P. Chandra in his Ph.D. thesis (P.U. 1977) titled "Investigation into the theory of operator and linear spaces" by the relation $E^3 = E$, where $E$ is a linear operator on a linear space $L$.

Clearly if $E$ is a projection, then it is also a trijection for

$$E^2 = E \Rightarrow E^3 = E^2 . E = E . E = E^2 = E.$$

Dr. Rajive Kumar Mishra in his Ph.D. thesis (J. P. U. 2010) titled "Study of linear operators and related topics in functional analysis" has defined a linear operator $E$ on a linear space $L$ to be tetrajection if $E^4 = E$.

From the concepts of projection, trijection, tetrajection, we define analogously a new type of operator $E$ called (4, 2)-jection if $E^4 = E^2$. It is a generalisation of projection operator in the sense that every projection is a (4, 2)-jection, but (4, 2)-jection is not a projection. This would be clear from the following examples of (4, 2)-jection gives below.

## 2. Examples of (4, 2)-jection :

We consider $C^2$, where $C$ is the set of all complex numbers. Let $z$ be an element in $C^2$. Thus let $z = (x, y)$, where $x, y$ belongs to $C$.

Let $E(z) = E(x, y) = (ax + by, cx + dy)$, where $a, b, c, d$ are scalars.

By calculation, we find that

$$E^2(x, y) = E(a_1 x + b_1 y, c_1 x + d_1 y)$$

Where,

$$a_1 = a^2 + bc$$

$$b_1 = ab + bd$$

$$c_1 = ca + cd$$

$$d_1 = bc + d^2$$

And also, $E^4(x, y) = (a_2x + b_2y, c_2x + d_2y)$

Where, $a_2 = a^4 + 3a^2bc + 2abcd + b^2c^2 + bcd^2$

$b_2 = a^3b + a^2bd + 2ab^2c + abd^2 + 2b^2cd + bd^3$

$c_2 = a^3c + a^2cd + 2abc^2 + acd^2 + 2bc^2d + cd^3$

$d_2 = a^2bc + 2abcd + b^2c^2 + 3bcd^2 + d^4$

If $E$ is a (4, 2)-jection then $E^4(x, y) = E^2(x, y)$.

Hence,

$$a_2 = a_1 \Rightarrow a^4 + 3a^2bc + 2abcd + b^2c^2 + bcd^2 = a^2 + bc \quad (2.1)$$

$$b_2 = b_1 \Rightarrow a^3b + a^2bd + 2ab^2c + abd^2 + 2b^2cd + bd^3 = ab + bd \quad (2.2)$$

$$c_2 = c_1 \Rightarrow a^3c + a^2cd + 2abc^2 + acd^2 + 2bc^2d + cd^3 = ca + cd \quad (2.3)$$

$$d_2 = d_1 \Rightarrow a^2bc + 2abcd + b^2c^2 + 3bcd^2 + d^4 = bc + d^2 \quad (2.4)$$

**2.I :** Now we consider in the following cases :

**Case (1) :** Let $a = b = 0$, then

from (2.1); $0 = 0$

from (2.2); $0 = 0$

from (2.3); $cd^3 = cd$

$cd(d^2 - 1) = 0$

$c = 0, d = 0, d = \pm 1$

And from (2.4); $d^4 = d^2$

$d^2(d^2 - 1) = 0$

$d = 0, d = \pm 1$

**Subcase (1.i) :** When $c \neq 0, d = 0$

Then we get $E(x, y) = (0, cx)$

$$E^2(x, y) = E(E(x, y)) = E(0, cx) = (0, 0)$$

$$E^2(x, y) \neq E(x, y)$$

Now, $E^3(x, y) = E(E^2(x, y)) = E(0, 0) = (0, 0)$

and, $E^4(x, y) = E(E^3(x, y)) = E(0, 0) = (0, 0)$

Clearly, $E^4(x, y) = E^2(x, y)$

Thus $E^4 = E^2$

Thus clearly in this case $E$ is neither projection nor trijection, but it is (4, 2)-jection.

**Subcase (1.ii) :** When $c = 0, d = 0$

Then we get $E(x, y) = (0, 0)$

$$E^2(x, y) = E(E(x, y)) = E(0, 0) = (0, 0) = 0$$

$$E^2(x, y) = E(x, y)$$

Now, $E^3(x, y) = E(E^2(x, y)) = E(0, 0) = (0, 0) = 0$

and, $E^4(x, y) = E(E^3(x, y)) = E(0, 0) = (0, 0) = 0$

Clearly, $E^4 = E^3 = E^2 = E = 0$ zero operator.

Thus in this case $E$ is projection, trijection, tetrajection, as well as (4, 2)-jection.

**Subcase (1.iii) :** When $c = 0, d = 1$

Then we get $E(x, y) = (0, y)$

$$E^2(x, y) = E(E(x, y)) = E(0, y) = (0, y)$$

$$E^2(x, y) = E(x, y)$$

Now, $E^3(x, y) = E(E^2(x, y)) = E(0, y) = (0, y)$

and, $E^4(x, y) = E(E^3(x, y)) = E(0, y) = (0, y)$

Clearly, $E^4 = E^3 = E^2 = E.$

Thus in this case $E$ is projection, trijection, tetrajection, as well as (4, 2)-jection.

**Subcase (1.iv) :** When $c = 0, d = -1$

Then we get $E(x, y) = (0, -y)$

$$E^2(x, y) = E(E(x, y)) = E(0, -y) = (0, y)$$

$$E^2(x, y) \neq E(x, y)$$

Now, $E^3(x, y) = E(E^2(x, y)) = E(0, y) = (0, -y) = E(x, y)$

and, $E^4(x, y) = E(E^3(x, y)) = E(0, -y) = (0, y) = E^2(x, y)$

Clearly, $E^4 = E^2 \neq E$ and $E^3 = E.$

Thus in this case $E$ is neither projection nor tetrajection, but it is trijection, as well as (4, 2)-jection.

**Subcase (1.v) :** When $c \neq 0, d = -1$

Then we get $E(x, y) = (0, cx - y)$

$$E^2(x, y) = E(E(x, y)) = E(0, cx - y) = (0, -cx + y)$$

$$E^2(x, y) \neq E(x, y)$$

Now, $E^3(x, y) = E(E^2(x, y)) = E(0, -cx + y) = (0, cx - y) = E(x, y)$

and, $E^4(x, y) = E(E^3(x, y)) = E(0, cx - y) = (0, -cx + y) = E^2(x, y)$

$$E^4(x, y) = E^2(x, y)$$

Thus in this case $E$ is neither projection nor tetrajection, but it is trijection, as well as (4, 2)-jection.

**Subcase (1.vi) :** When $c \neq 0, d = 1$

Then we get $E(x, y) = (0, cx + y)$

$$E^2(x, y) = E(E(x, y)) = E(0, cx + y) = (0, cx + y)$$

$$E^2(x, y) = E(x, y)$$

Now, $$E^3(x, y) = E(E^2(x, y)) = E(0, cx + y) = (0, cx + y) = E(x, y)$$

and, $$E^4(x, y) = E(E^3(x, y)) = E(0, cx + y) = (0, cx + y) = E^2(x, y)$$

$$E^4(x, y) = E^2(x, y).$$

Thus in this case $E$ is projection, trijection, tetrajection, as well as (4, 2)-jection.

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