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Euclidean distance approaches for intuitionistic Fuzzy sets and related Fuzzy constructs

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Abstract:

This paper introduces the Euclidean Fuzzy Information Measure (EFIM), a novel index designed to enhance the analysis of intuitionistic fuzzy sets (IFS) through a Euclidean-based framework. The EFIM offers a framework for reinterpreting normalized geometric separation from an information ideal, establishing it as a standardized measure of residual uncertainty, where proximity to this ideal correlates with enhanced informational density. This study presents a precise mathematical formulation of the EFIM for single-valued intuitionistic fuzzy sets (IFSs), extends the framework to encompass interval-valued and other generalized fuzzy contexts, and methodically examines *essential properties of an information measure-specifically boundedness, monotonicity in relation to proximity, and continuity in membership parameters-while providing a detailed numerical example employing digit-by-digit arithmetic and a compact diagnostic application with three symptom indicators for practical aggregation and interpretation. In contrast to traditional Euclidean ranking techniques and established fuzzy entropy formulations, EFIM not only maintains geometric intuition but also offers a normalized [0,1] scale and an information oriented interpretation, effectively addressing critiques regarding the inadequacy of raw distances as suitable measures of information. The paper delineates essential practical adaptations, including the weighting of membership components, the establishment of alternative reference points, and the development of projection methods for interval or hesitant values, all aimed at enhancing domain calibration within multi-criteria decision-making (MCDM), medical diagnostics, and spatial evaluation tasks. Recognizing limitations involves carefully selecting a reference point, addressing moments of hesitation, and emphasizing the necessity for empirical validation. In summary, we propose a comprehensive roadmap for future research that includes benchmarking the EFIM against established entropies and distance metrics with real datasets, integrating EFIM within TOPSIS-like frameworks, and exploring its generalization to accommodate complex or hesitant fuzzy structures. The objective is to establish a straightforward and adaptable framework that effectively integrates distance-based ranking methodologies with information theoretic representations in fuzzy systems.*

Keywords: Euclidean distance, fuzzy information, intuitionistic fuzzy set, information index, decision support

1. Introduction:

Intuitionistic fuzzy sets (IFS) extend classical fuzzy sets by explicitly modelling membership, non-membership, and the residual hesitation. Researchers have widely used distance metrics- commonly Euclidean forms on IFS for ranking and decision problems (e.g., medical diagnosis, MCDM, GIS), because geometric distances are intuitive and computationally straightforward (Chan & Abdullah, 2020; Xing, Xiong, & Liu, 2016). Separately, fuzzy information measures and entropies formalize what it means for a fuzzy object to be "uncertain" or to contain information

(Kumar, 2020; Ohlan, 2016). Several authors have warned that raw distances lack the bounded interpretation and semantic clarity expected from an information index (Arman et al., 2022; Hosein Arman et al., 2022). Motivated by this gap, we design EFIM: a normalized Euclidean-based index that retains geometric intuition while providing an interpretable [0, 1] information scale.

Contributions: (1) a clear definition of EFIM for single-valued IFS and a simple extension recipe for interval/complex forms; (2) proofs of essential properties; (3) a fully worked numeric example with step-by-step arithmetic; (4) a short applied illustration and suggestions for future validation.

2. Related work:

Euclidean distance approaches for IFS and related fuzzy constructs have been used for ranking and practical applications (Chan & Abdullah, 2020; Xing et al., 2016; Manonmani & Suganya, 2018). Reviews and surveys of fuzzy information measures establish desirable axioms for entropies and relative information indices (Kumar, 2020; Ohlan, 2016). Modern studies extend distance measures to complex or hesitant fuzzy contexts (Khan et al., 2024). Critical examinations of ED usage in fuzzy TOPSIS and similar frameworks highlight the need for normalization and semantic care (Arman et al., 2022). We draw on these strands to build a normalized, information-oriented Euclidean index.

3. Preliminaries:

Let $X = \{x_1, x_2, ..., x_m\}$. An intuitionistic fuzzy set A assigns to each $x \in X$ a membership $\mu_A(x) \in [0, 1]$ and non membership $\nu_A(x) \in [0, 1]$ with $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. The hesitation (indeterminacy) is $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$. We represent an element by the 3-tuple $v = (\mu, \nu, \pi)$. For two 3-tuples $v_1 = (\mu_1, \nu_1, \pi_1)$ and $v_2 = (\mu_2, \nu_2, \pi_2)$, the Euclidean distance is

$$d_E(v_1, v_2) = \sqrt{[(\mu_1 - \mu_2)^2 + (\nu_1 - \nu_2)^2 + (\pi_1 - \pi_2)^2]}$$

4. Definition of EFIM:

Choose an information ideal $I = (1,0,0)$ (maximum membership, zero non-membership and hesitation). For a single element $v = (\mu, \nu, \pi)$ define the local normalized distance $\delta(v) = d_E(v, I)$.

Because the admissible v satisfy $\mu + \nu + \pi = 1$ and all components lie in $[0, 1]$, the maximum possible distance from I occurs within the feasible region but is bounded by

1. Thus we normalizes by 1 and define the local EFIM value

$$\text{EFIM}(v) = 1 - \delta(v) = 1 - \sqrt{[(1-\mu)^2 + \nu^2 + \pi^2]}$$

For a finite IFS A on X the global EFIM is the arithmetic mean of local scores.

$$\text{EFIM}(A) = (1/m)\sum_{i=1}^{m} \text{EFIM}(v_i) = 1 - (1/m)\sum_{i=1}^{m} \sqrt{[(1-\mu_i)^2 + \nu_i^2 + \pi_i^2]}$$

If desired, replace the unweighted mean by a weighted average where $w_i \geq 0$, $\sum w_i = 1$ to reflect importance of elements.

Rationale: EFIM maps Euclidean proximity to a [0, 1] information index: higher EFIM indicates closer to the informational ideal and thus greater information content (less residual uncertainty).

5. Theoretical properties:

Proposition 1 (Boundedness): For any IFS A, $0 \leq \text{EFIM}(A) \leq 1$.

Proof: (Sketch): Each term $\text{EFIM}(v) = 1 - \delta(v)$. Satisfies $0 \leq \delta(v) \leq 1$ under normalization, so $0 \leq \text{EFIM}(v) \leq 1$. Average of terms remains in [0, 1].

Proposition 2 (Monotonicity): If element-wise distance of A are no greater than those of B (i.e., $\forall\ i$: $\delta_A(x_i) \leq \delta_B(x_i)$), then $\text{EFIM}(A) \geq \text{EFIM}(B)$.

Proof: Immediate from the EFIM formula as it is decreasing in δ.

Continuity: EFIM is continuous in μ, ν, π because it is composed of continuous algebraic operations.

These properties align EFIM with common axiomatic expectations for information measures (Kumar, 2020).

6. Extensions to other fuzzy types:

- **Interval-valued IFS:** Replace triplet values by their mid-point or use center of gravity to compute EFIM.

- **Hesitant/complex fuzzy values:** Use embedding or projection to map the structure to a finite vector of representative coordinates, then compute a normalized Euclidean distance.

- **Weighted reference:** Replace I by a domain-specific reference $I_W = (\mu^*, \nu^*, \pi^*)$ and normalize by the maximal feasible distance to I_W.

7. Worked numeric example (digit-by-digit arithmetic):

Example 1: Two single values (detailed arithmetic):

Given:

A : $\mu_A = 0.70$, $\nu_A = 0.20$. Then $\pi_A = 1 - 0.70 - 0.20 = 0.10$.

B : $\mu_B = 0.40$, $\nu_B = 0.40$. Then $\pi_B = 1 - 0.40 - 0.40 = 0.20$.

Compute $\text{EFIM}(v) = 1 - \sqrt{[(1-\mu)^2 + \nu^2 + \pi^2]}$

For A:

1. $1 - \mu_A = 1 - 0.70 = 0.30$.
2. $(1 - \mu_A)^\wedge 2 = 0.30 \times 0.30 = 0.090$.
3. $\nu_A{}^\wedge 2 = 0.20 \times 0.20 = 0.040$.
4. $\pi_A{}^\wedge 2 = 0.10 \times 0.10 = 0.010$.
5. Sum squares: $0.090 + 0.040 + 0.010 = 0.140$.
6. Square root: $\sqrt{0.140} \approx 0.3741657$.
7. $\text{EFIM}(A) = 1 - 0.3741657 = 0.6258343$.

Round to four decimals: $\text{EFIM}(A) \approx 0.6258$.

For B:

1. $1 - \mu_B = 1 - 0.40 = 0.60$.
2. $(1 - \mu_B)^\wedge 2 = 0.60 \times 0.60 = 0.360$.
3. $\nu_B{}^\wedge 2 = 0.40 \times 0.40 = 0.160$.

4. π_B^ 2 = 0.20 × 0.20 = 0.040.
5. Sum squares: 0.360 + 0.160 + 0.040 = 0.560.
6. Square root: $\sqrt{0.560} \approx 0.7483315$.
7. EFIM(B) = 1 - 0.7483315 = 0.2516685.

Round to four decimals: EFIM(B) ≈ 0.2517.

Interpretation: A (0.6258) > B (0.2517). A is closer to the ideal and therefore carries more information per EFIM.

Example 2: Compact diagnostic scenario (three indication):

Symptom indicators produce IFS triplets:

$$x1 : (\mu_1 = 0.90,\ \nu_1 = 0.05) \rightarrow \pi_1 = 1 - 0.90 - 0.05 = 0.05$$

$$x2 : (\mu_2 = 0.60,\ \nu_2 = 0.30) \rightarrow \pi_2 = 0.10$$

$$x3 : (\mu_3 = 0.40,\ \nu_3 = 0.50) \rightarrow \pi_3 = 0.10$$

Compute EFIM for each then average.

For x1:

1. $1 - \mu_1 = 0.10$. Square: $0.10^2 = 0.010$.
2. $\nu_1^2 = 0.05^2 = 0.0025$.
3. $\pi_1^2 = 0.05^2 = 0.0025$.
4. Sum = 0.010 + 0.0025 + 0.0025 = 0.0150.
5. $\sqrt{0.0150} \approx 0.1224745$.
6. EFIM(x1) = 1 - 0.1224745 = 0.8775255 ≈ 0.87753.

For x2:

1. $1 - \mu_2 = 0.40$. Square: $0.40^2 = 0.160$.
2. $\nu_2^2 = 0.30^2 = 0.090$.

3. $\pi_2^2 = 0.10^2 = 0.010$.
4. Sum = 0.160 + 0.090 + 0.010 = 0.260.
5. $\sqrt{0.260} \approx 0.5099019$.
6. EFIM(x2) = 1 - 0.5099019 = 0.4900981 ≈ 0.49010.

For x3:

1. $1 - \mu_3 = 0.60$. Square: $0.60^2 = 0.360$.
2. $\nu_3^2 = 0.50^2 = 0.250$.
3. $\pi_3^2 = 0.10^2 = 0.010$.
4. Sum = 0.360 + 0.250 + 0.010 = 0.620.
5. $\sqrt{0.620} \approx 0.7874016$.
6. EFIM(x3) = 1 - 0.7874016 = 0.2125984 ≈ 0.21260 (rounded).

Aggregate EFIM (unweighted mean):

1. Sum EFIM values: 0.8775255 + 0.4900981 + 0.2125984 =
   - First add 0.8775255 + 0.4900981 = 1.3676236.
   - Then add 0.2125984 → total = 1.5802220.

   Here 1.5802220 is consistent with these precise values.
2. Divide by 3: 1.5802220 / 3 = 0.52674066 ≈ 0.52674 (rounded).

Interpretation:

The aggregated EFIM = 0.5267 indicates a moderate information level across symptoms; x1 contributes most to information (highest EFIM).

8. Comparison and discussion:

EFIM provides a normalized interpretable scale unlike raw Euclidean distance used only for ordering (Arman et al., 2022). Compared to fuzzy entropy indices, EFIM is geometrically founded and computationally straightforward, but it is not designed to replace entropy's probabilistic interpretations; instead it offers a pragmatic, comparable index that can be embedded into decision pipelines (Chan & Abdullah, 2020; Kumar, 2020). EFIM's simplicity facilitates extension to IVIFS and hesitant scenarios via projection; however, the choice of representative projection affects values and should be reported in empirical studies (Manommani & Suganya, 2018; Khan et al., 2024).

9. Limitations and future directions:

- **Reference choice:** We used $I = (1, 0, 0)$; alternative reference points (domain-specific ideals) could be more appropriate in some applications.
- **Hesitation handling:** Different schemes for hesitant values produce different representative points; research should compare projection strategies.
- **Empirical validation:** Benchmarking EFIM on real datasets (medical records, MCDM benchmarks, GIS layers) is essential to quantify benefits vs. existing entropies and distance techniques.
- **Integration:** Embedding EFIM into ranking frameworks (e.g., TOPSIS variants) and studying sensitivity to weighting are promising next steps (Xing et al., 2016; Arman et al., 2022).

10. Conclusion:

We introduced EFIM, a normalized Euclidean-based information index for IFS that merges geometric proximity intuition with an interpretable $[0, 1]$ information scale. EFIM is easy to compute, satisfies basic axioms (boundedness, monotonicity, continuity), and extends to interval/complex fuzzy types through representative mapping.

The worked examples illustrate exact computation and interpretation. Next steps are empirical benchmarking, weighted/parameterized generalizations, and application studies in diagnosis, MCDM, and GIS.

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