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New Results on Laplace Transform and its Applications

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Abstract:

In this paper, we applied new results on Laplace Transform for solving the Partial Differential Equations of triple variables, that is, Wave equation & Heat equation. Through this methodology we tried to prove that this method is very effective & convenient for solving Partial Differential Equation.

1.1 Introduction and Preliminaries:

The Concept of Laplace Transform are used in the form of wave equation in Mathematical Physics, Applied Mathematics and Engineering field. There it is known that there are two types of equation one is Homogeneous equation that has Content Coefficient with many classical solution such as separation of variables by G.L. Lamb [1995], the method of characteristic result by V.T. Single [1980] and C. Constanda [2000] single Laplace of fourier transform by D.G Duffy [2004] at non homogeneous equation with constant coefficient solved by double Laplace transform by A. Babakhani and R.S. Datiya [2001]; Hassan Eltayeb and Adem Kilicman [2013]. In this study, we use triple Laplace transform to serve equation and partial differential equation. We use the method by Hassan Eltayeb and Adem Kilicman (2013), where they extended two-dimensional convention theorem.

First of all we recall the following results given by Kilicman and Godain [2010]. The double Laplace transform is define by

$$L_xL_t[f(x,s)] = F(p,s) = \int_0^\infty e^{-ps} \int_0^\infty e^{-ps} \; f(x,t)dtdx \qquad (1.1.1)$$

Where $x, t > 0$ and p, s are Complex and the first-order partial derivative is defined follows

$$L_xL_t \; \left[\frac{\partial[f(x,t)]}{\partial x}\right] = p\,F(p,s) - F(0,s) \qquad (1.1.2)$$

Double Laplace transform for 2nd order partial derivative with respect to x is given by

$$L_{xx}\left[\frac{\partial^2[f(x,t)]}{\partial^2 x}\right] = p^2\,F(p,s) - p\,F(0,s)\text{-}\frac{\partial[F(0,t)]}{\partial x} \qquad (1.1.3)$$

and double Laplace transform for second partial derivative with respect to t similarly on the previous is given by

$$L_{tt}\left[\frac{\partial^2[f(x,t)]}{\partial^2 t}\right] = s^2\,F(p,s) - s\,F(p,0)\text{-}\frac{\partial[F(0,t)]}{\partial x} \qquad (1.1.4)$$

In a similar manner, the double Laplace transform of a mixed partial derivative can be derived from a single Laplace transform for as

$$L_xL_t \; \left[\frac{\partial^2[f(x,t)]}{\partial x \partial t}\right] = psF(p,s) - pF(p,0)\text{-}\,sF(0,s) - F(0,0) \qquad (1.1.5)$$

Now the triple Laplace transform is define by

$$L_xL_yL_t[f(x,y,s)] = F(p,q,s)$$

$$= \int_0^\infty e^{-ps} \int_0^\infty e^{-qs} \int_0^\infty e^{-ts} \; f(x,y,t)dtdydx \quad (1.1.6)$$

Where $x, y, t > 0$ and p, q, s are complex number and the first-order partial derivative as follow

$$L_xL_yL_t \left[\frac{\partial[f(x,t)]}{\partial x}\right] = pF(p,q,s) \text{-} F(0,q,s) - F(0,q,s) \quad (1.1.7)$$

Triple Laplace transform for 3[rd] partial derivative with respect to x is given by

$$L_{xxx}\left[\frac{\partial^3[f(x,y,t)]}{\partial^3 x}\right] = p^3\,F(p,q,s) - p^2\,F(0,q,s) - p\,F(0,q,s)\frac{\partial[F(0,q,s)]}{\partial x} \quad (1.1.8)$$

And Triple Laplace transform for 3[rd] partial derivative with respect to y is given by

$$L_{yyy}\left[\frac{\partial^3[f(x,y,t)]}{\partial^3 y}\right] = q^3\,F(p,q,s) - q^2\,F(p,0,s) - q\,F(p,0,s)\frac{\partial[F(p,0,s)]}{\partial y} \quad (1.1.9)$$

Similar to t with respect 3[rd] order partial derivative with respect to t is given by

$$L_{ttt}\left[\frac{\partial^3[f(x,y,t)]}{\partial^3 t}\right] = s^3\,F(p,q,s) - s^2\,F(p,qz,0) - s\,F(p,q,0)\frac{\partial[F(p,q,0)]}{\partial t} \quad (1.1.10)$$

In the same way, the triple Laplace transform of a mixed partial derivative can be a define as

$$L_xL_yL_t \left[\frac{\partial^3[f(x,y,t)]}{\partial x\partial y\partial t}\right] = pqsF(p,q,s) \text{-} pF(p,0,0)$$

$$- qF(0,q,0) \text{-} sF(0,0,s) \text{-} F(0,0,0) \quad (1.1.11)$$

1.2 Main Theorem:

In this section, we develop some theorem by the definition of above concepts which of extension of the Hassan Eltayeb and Adem Kilicman [2013].

Theorem (1): If at the point (p, q, s) the integral

$$F_1(p,q,s) = \int_0^\infty\int_0^\infty\int_0^\infty e^{-px-qy-sz}\,F_1(p,q,s)dx\,dydz \quad (1.2.1)$$

is convergent and the integral

$$F_2(p,q,s) = \int_0^\infty\int_0^\infty\int_0^\infty e^{-px-qy-sz}\,F_2(p,q,s)dx\,dydz \quad (1.2.2)$$

is absolutely convergent then

$$F(p,q,s) = F_1(p,q,s).F_2(p,q,s)$$

in the Laplace transform of the function

$$f(x,y,z)=\int_0^x\int_0^y\int_0^z F_1(x-\alpha,y-\beta,z-\gamma)\,F_2(\alpha,\beta,\gamma)d\alpha d\beta d\gamma \tag{1.2.3}$$

are the integral

$$F(p,q,s)=\int_0^\infty\int_0^\infty\int_0^\infty e^{-px-qy-sz}\,F(p,q,s)dx\,dydz \tag{1.2.4}$$

is convergent at the point (p,q,s).

Proof: The uniqueness and existence of Laplace transform first of all, let $f(x,y,t)$ be a continuous function, which is of exponential order for some $a,b,c\in R$. Consider

$$\max_{\substack{t>0\\x>0\\y>0}}\frac{|f(x,y,t)|}{e^{ax+by+ct}}<\infty \tag{1.2.5}$$

In this case, the result is Triple Laplace transform of

$$F(p,q,s)=\int_0^\infty\int_0^\infty\int_0^\infty e^{-st-px-qy}\,F(p,q,s)dx\,dydt \tag{1.2.6}$$

exists for all $p>a$, $q>b$ and $s>c$ all functions in this study are assumed to be exponential order of the $f(x,y,t)$ can be uniquely recovered from $F(p,q,s)$.

$$F(p,q,s)=\int_0^\infty\int_0^\infty\int_0^\infty e^{-px-qy-sz}\,F(x,y,z)dx\,dydz$$

Theorem (2): Let $f(x,y,t)$ be the continuous function defined for $x,y,t>0$ Laplace transform $F(p,q,s)$ and $G(p,q,s)$ the $f(x,y,t)=g(x,y,t)$.

Proof: If α, β and γ sufficiently large then the integral representation by

$$\begin{aligned}f(x,y,t)&=\frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty}e^{px}\left\{\frac{1}{2\pi i}\int_{\beta-i\infty}^{\beta+i\infty}e^{qy}\left\{\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}e^{st}\,F(p,q,s)\right\}\right\}dsdpdq\\&=\frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty}e^{px}\left\{\frac{1}{2\pi i}\int_{\beta-i\infty}^{\beta+i\infty}e^{qy}\left\{\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}e^{st}\,G(p,q,s)\right\}\right\}dsdpdq\\&=g(x,y,t)\end{aligned} \tag{1.2.7}$$

Theorem (3): A function $f(x,y,t)$ which is continuous and satisfies the growth condition defined as

$$F(x,y,t)=\lim_{\substack{x\to\infty\\y\to\infty\\z\to\infty}}\frac{(-1)^{l+m+n}}{l!m!n!}\left(\frac{l}{m}\right)^{l+1}\left(\frac{m}{y}\right)^{m+1}\left(\frac{n}{z}\right)^{n+1}\varphi^{l+m+n}\left(\frac{l}{x}\quad\frac{m}{y}\quad\frac{n}{z}\right) \tag{1.2.8}$$

Where φ^{l+m+n} is derived by $(l+m+n)$th mixed potential derivative of $F(p,q,s)$.

$$\varphi^{l+m+n} = \frac{\partial^{l+m+n} F^{(p,q,s)}}{\partial p^l \partial q^m \partial s^n} \text{ for } x, y, t > 0$$

Proof: Prove of the theorem as similar to previous theorem obtains in term of $F(p,q,s)$.

Example : Let $f(x, y, t) = e^{-ax-by-cz}$

The Laplace transform is easily found to be as follows

$$F(p,q,s) = \frac{1}{(p+q)(q+b)(s+c)} \tag{1.2.9}$$

It is above simple to verify that

$$\frac{\partial^{l+m+n} F^{(p,q,s)}}{\partial p^l \partial q^m \partial s^n} = l!m!n!(-1)^{l+m+n} (p+q)^{-l-1} (q+b)^{-m-1} (s+c)^{-n-1} \tag{1.2.10}$$

$\dfrac{\partial^{l+m+n} F^{(p,q,s)}}{\partial p^l \partial q^m \partial s^n}$ into theorem (3) given the following result

$$F(x, y, t) = \lim_{\substack{x \to \infty \\ y \to \infty \\ z \to \infty}} \frac{l^{l+1} m^{m+1} n^{n+1}}{x^{l+1} y^{m+1} z^{n+1}} \left(a + \frac{l}{x}\right)^{-l-1} \left(b + \frac{m}{y}\right)^{-m-1} \left(c + \frac{n}{t}\right)^{-n-1} \tag{1.2.11}$$

$$= \lim_{\substack{x \to \infty \\ y \to \infty \\ z \to \infty}} \left(1 + \frac{ax}{l}\right)^{-l-1} \left(1 + \frac{by}{m}\right)^{-m-1} \left(1 + \frac{ct}{n}\right)^{-n-1}$$

The last limit is easy to evaluate take the natural log of both side and write the result in the form

$$[l_n(1+a_n/l)/(1/l+1)] - [l_n(1+b_y/m)/(1/m+1)] - [l_n(1+c_t/n)/(1/t+1)]$$

Using the L. hospital rule that the indeterminate from approaches $-ax - by - ct$.

The continuity of the natural logarithm shows that

In $(f(x, y, t)) = -ax - by - cz$

then $f(x, y, t) = e^{-ax-by-cz}$

1.3 Properties of triple Laplace Transform:

In this section, we consider some of the proposition of the triple Laplace transform that will be enable us to find further transform pairs $\{f(x, y, t), f(p, q, s)\}$ without having to Compute consider the following

(1) $F(p+e, q+f, s+g) = L_xL_yL_t[e^{-ex-fy-gt} f(x, y, t)](p, q, s)$

(2) $1/k\, F[p/\alpha, q/\beta, s/\gamma] = L_xL_yL_t[f(\alpha x, \beta y, \gamma t)]$, where $k = \alpha\beta\gamma$

$$(3)\ \frac{\partial^{l+m+n} F^{(p,q,s)}}{\partial p^l \partial q^m \partial s^n} = L_xL_yL_t[(-1)^{l+m+n} x^l y^m t^n f(x, y, t)](p, q, s) \qquad (1.3.1)$$

Proof:

$$(1)\ L_xL_yL_t[e^{-ex-fy-gt} f(x, y, t)](p, q, s) = \int_0^\infty\int_0^\infty\int_0^\infty e^{-ex-fy-gt}\, e^{-st-qy-px} f(x,y,t)dt\,dy\,dx$$

$$= \int_0^\infty e^{-ex-fy-gt}[\int_0^\infty\int_0^\infty e^{-ex-fy-gt} f(x,y,t)dt\,dy\,dx] \quad (1.3.2)$$

We calculate the integral inside brackets as

$$\int_0^\infty e^{-ex-fy-gt} f(x,y,t)dt = f(x,y,s+c) \qquad (1.3.3)$$

By substituting we get,

$$L_xL_yL_t[e^{-ex-fy-gt} f(x,y,s+c)](p, q, s) = \int_0^\infty\int_0^\infty e^{-ex-fy-gt}(x,y,s+c)dx\,dy$$

$$= F(p+e, q+f, s+c) \qquad (1.3.4)$$

We have

$$L_xL_yL_t = [f(\alpha x, \beta y, \gamma t)](p, q, s) = \int_0^\infty\int_0^\infty\int_0^\infty e^{-st-qy-px} f(\alpha x, \beta y, \gamma t)dx\,dy\,dt$$

$$= \int_0^\infty e^{-st}\Big(\int_0^\infty {}^{-qy}\Big(\int_0^\infty {}^{-px} f(\alpha x, \beta y, \gamma t)\,dx\Big)dy\Big)dt$$

$$= \int_0^\infty e^{-st} \frac{1}{\alpha} F\left(\frac{p}{\alpha}, \frac{q}{\beta}, \gamma t\right) dt = \frac{1}{\alpha\beta} F\left(\frac{p}{\alpha}, \frac{q}{\beta}, \frac{s}{\gamma}\right) \quad (1.3.5)$$

The last property from the definition of Triple transform

$$F(p, q, s) = \int_0^\infty \int_0^\infty \int_0^\infty e^{-st-qy-px} f(x,y,t) dxdydt \quad (1.3.6)$$

So that

$$\frac{\partial^{l+m+n} F^{(p,q,s)}}{\partial p^l \partial q^m \partial s^n} = \frac{\partial^{l+m+n}}{\partial p^l \partial q^m \partial s^n} \int_0^\infty \int_0^\infty \int_0^\infty e^{-st-qy-px} f(x,y,t) dxdydt \quad (1.3.7)$$

By the convergence properties of the important Integral, we can interchange the operation differentiation and information and differentiate with respect to p, q and s under the given. Thus

$$\frac{\partial^{l+m+n} F^{(p,q,s)}}{\partial p^l \partial q^m \partial s^n} = \int_0^\infty \frac{\partial^n}{\partial s^n} e^{-st} \left\{ \frac{\partial^m}{\partial q^m} e^{-qy} \left\{ \frac{\partial^l}{\partial p^l} e^{-px} f(x,y,t) \right\}\right\} dt \quad (1.3.8)$$

which on carrying out the repeated differentiation with respect to p, q, s is given as

$$\frac{\partial^{l+m+n} F^{(p,q,s)}}{\partial p^l \partial q^m \partial s^n} = (-1)^{l+m+n} \int_0^\infty \int_0^\infty \int_0^\infty x^l y^m t^n e^{-st-qy-px} f(x,y,t) dxdydt$$

$$= (-1)^{l+m+n} L_x L_y L_t \, [x^l y^m t^n f(x,y,t)](p,q,s) \quad (1.3.9)$$

Proof of theorem (3): Let first defined the set of function depending on the parameters l, m, n as

$$g_{l,m,n}(x,y,t) = \frac{l^{l+1} m^{m+1} n^{n+1}}{l! m! n!} x^l y^m t^n e^{-lx-my-nt}$$

So $\int_0^\infty \int_0^\infty \int_0^\infty g_{l,m,n}(x,y,t) dxdydt = 1$ (1.3.10)

$$\lim_{\substack{x\to\infty \\ y\to\infty \\ z\to\infty}} \int_0^\infty \int_0^\infty \int_0^\infty g_{l,m,n}(x,y,t)\, \phi(x,y,t) dxdydt = \phi(1,1,1)$$

Where $\phi(x,y,t)$ is any Continuous function let us develop its Laplace transform for as a function p, q, s by

$$L_xL_yL_t[x^ly^mt^nf(x,y,t)](p,q,s)$$

Now, we define the function

$$\phi(x,y,t) = f(xx_0, yy_0, tt_0) \text{ and by property (2) we have}$$

$$L_xL_yL_t[\varphi(x,y,t)](p,q,s) = L_xL_yL_t[f(xx_0, yy_0, tt_0)](p,q,s)$$

$$= \frac{1}{x_0y_0z_0}\left(\frac{p}{x_0}\frac{q}{y_0}\frac{s}{z_0}\right) \tag{1.3.11}$$

We apply properly (3), On evaluating partial derivation of mixed $l+m+n$ type function $F(p,q,s)$ at points

$$p = 1/x,\ q = m/y \text{ and } s = n/t \text{ as follows}$$

$$\frac{\partial^{l+m+n}F^{(p,q,s)}}{\partial p^l\partial q^m\partial s^n} = L_xL_yL_t[\varphi(x,y,t)](p,q,s)$$

$$= \frac{1}{x_0^{l+1}y_0^{m+1}t_0^{n+1}}\frac{\partial^{l+m+n}F^{(p,q,s)}}{\partial p^l\partial q^m\partial s^n}F\left(\frac{p}{x_0}\frac{q}{y_0}\frac{s}{z_0}\right) \tag{1.3.12}$$

Let $\phi(x,y,t) = e^{-px-qy-st}\varphi(x,y,t)$ by using (1.3.10), we get

$$\phi(1,1,1) = e^{-p-q-s}\varphi(x,y,t) = e^{-p-q-s}f(x_0y_0z_0)$$

$$\lim_{\substack{l\to\infty\\ m\to\infty\\ n\to\infty}}\frac{l^{l+1}m^{m+1}n^{n+1}}{l!m!n!}\int_0^\infty\int_0^\infty\int_0^\infty x^ly^mt^ne^{-px-qy-st}e^{-lx-my-nt}\varphi(x,y,t)dxdy \tag{1.3.13}$$

$$\lim_{\substack{l\to\infty\\ m\to\infty\\ n\to\infty}}\frac{l^{l+1}m^{m+1}n^{n+1}}{l!m!n!}L_xL_yL_t[x^ly^mt^ne^{-lx-my-nt}]\varphi(x,y,t)(p,q,s) \tag{1.3.14}$$

By using the expressions properties (1) and (3) of the triple Laplace transform (1.3.13) and the definition of $\varphi(x,y,t)$. We get

$$L_xL_yL_t[x^ly^mt^ne^{-lx-my-nt}]\varphi(x,y,t)(p,q,s)$$

$$=\frac{(-1)^{l+m+n}\partial^{l+m+n}}{\partial p^l\partial q^m\partial s^n}\{L_xL_t\{e^{-lx-my-nt}\varphi(x,y,t)\}(p,q,s)\}$$

$$=\frac{(-1)^{l+m+n}\partial^{l+m+n}}{\partial p^l\partial q^m\partial s^n}[L_xL_yL_t[e^{-lx-my-nt}\varphi(x,y,t)](p,q,s)]$$

$$=(-1)^{l+m+n}\frac{1}{z}\frac{\partial^{l+m+n}}{\partial p^l\partial q^m\partial s^n}\left[F\left(\frac{p+l}{x_0},\frac{q+m}{y_0},\frac{s+n}{t_0}\right)\right] \quad (1.3.15)$$

Where $\frac{1}{z}=\frac{1}{x_0^{l+1}y_0^{m+1}t_0^{n+1}}$ from (1.3.14) and (1.3.15)

With $f(x_0y_0,t)=e^{p+q+s}\phi(1,1,1)$ we have $f(x_0y_0,t)$

$$e^{p+q+s}\lim_{\substack{l\to\infty\\ m\to\infty\\ n\to\infty}}\frac{l^{l+1}m^{m+1}n^{n+1}}{l!m!n!}\left(\frac{l}{x_0}\right)^{l+1}\left(\frac{m}{y_0}\right)^{m+1}\left(\frac{n}{t_0}\right)^{n+1}$$

$$\times\frac{\partial^{l+m+n}}{\partial p^l\partial q^m\partial s^n}\left[F\left(\frac{p+l}{x_0},\frac{q+m}{y_0},\frac{s+n}{t_0}\right)\right]$$

for any pqs, the statement is therefore is actually join the special case for $p=0$, $q=0$, $s=0$.

Example: Find the triple Laplace transform for a regular generated function

$$F(x,y,t)=H(t)*H(y)*H(x)\,.\,l_n(t)l_n(y)l_n(x) \quad (1.3.16)$$

$$\left[\frac{\partial^3[f(x,y,t)]}{\partial x\partial y\partial t}\right]=\left[\frac{\partial^3}{\partial x\partial y\partial t}\right]=H(t)*H(y)*H(x)$$

$$l_n(t)l_n(y)l_n(x)=\text{pt}\left[\frac{H(t)*H(y)*H(x)}{xyt}\right]$$

Where $H(x,y,t)=H(t)\times H(y)\times H(x)$ is a main side function and X is tensor product.

The triple Laplace transform with respect to x, y, t and 1 becomes

$$L_xL_yL_t[f(x, y, t)] = \int_0^\infty e^{-px} l_n(x) \int_0^\infty e^{-qy} l_n(y) \int_0^\infty e^{-st} l_n(t) dt\, dy\, dx$$

$$= -\frac{1}{sq} \int_0^\infty e^{-px} l_n(x)[\gamma^2 + l_n p\, l_n q\, l_n s] \qquad (1.3.17)$$

Where γ^2 is Euler's constant [6]. Then

$$L_xL_yL_t[f(x, y, t)] = -\frac{1}{spq}[\gamma^2 + l_n p\, l_n q\, l_n s] \text{ where Re} > 0 \qquad (1.3.18)$$

Triple Laplace transformation and (1.3.17) with respect to x, y, t is obtained as follows

$$L_xL_yL_t\left[\frac{\partial^3[f(x, y, t)]}{\partial x \partial y \partial t}\right] = L_xL_yL_t[H(t)H(y)H(x)\, l_n(t)\, l_n(y)\, l_n(x)]$$

$$= pqs\left[\frac{1}{spq}[\gamma^2 + l_n(p)\, l_n(q)\, l_n(s)]\right]$$

$$= \gamma^2 + l_n(p)\, l_n(q)\, l_n(s) \qquad (1.3.19)$$

Example: Let us now find triple Laplace transform of the function

$$(x^\alpha + y^\beta + z^\gamma) = H(t) \times H(y) \times H(x) x^\alpha y^\beta z^\gamma \quad \text{where } \alpha\beta\gamma \neq -1, -2, ...$$

Since $x^\alpha + y^\beta + z^\gamma \in L$

The triple Laplace transform of the function

$(x^\alpha + y^\beta + z^\gamma) = H(x) \times H(y) \times H(t) x^\alpha y^\beta z^\gamma$ is given by

$$L_xL_yL_t[(x^\alpha + y^\beta + z^\gamma)] = \int_0^\infty x^\alpha e^{-px} \int_0^\infty y^\beta e^{-qy} \int_0^\infty z^\gamma e^{-st}\, dt\, dy\, dx \qquad (1.3.20)$$

On partial

$$U = p_x, \quad V = q_y, \quad W = s_t \quad \text{for } p, q, t > 0$$

$$L_xL_yL_t[(x^\alpha + y^\beta + z^\gamma)] = \int_0^\infty u^\alpha e^{-u} \int_0^\infty v^\beta e^{-v} \int_0^\infty w^\gamma e^{-w} dw\, du\, dv$$

$$= \frac{1}{p^{\alpha+1} q^{\beta+1} s^{\gamma+1}} \Gamma(\alpha+1)\Gamma(\beta+1)\Gamma(\gamma+1) \qquad (1.3.21)$$

Consider in the several telegraph equation of the following form

$$u_{xxx}=u_{xxx}+u_{tt}+u_t+u+f(x,y,t) \tag{1.3.22}$$

With boundary condition

$$U(0,y,t)=f_1(t),\ u_x(0,y,t)=f_2(t),\ u_y(0,y,t)=f_3(t) \tag{1.3.23}$$

And initial condition

$$u(x,y,0)=g_1(x),\ u_y(x,y,0)=g_2(x),\ u_x(x,y,0)=g_3(x) \tag{1.3.24}$$

We apply three Laplace transform and we single Laplace transform for (1.3.23) and (1.3.24) after taking three inverse Laplace transform we get the solution of (1.3.22) in the form

$$U(x,t)=L_p^{-1}L_q^{-1}L_s^{-1}\left[\frac{F(p,s)+p^2F_1(s)+pF_2(s)+F_3(s)}{p^3-s^3-s^2-s-1}\ \frac{G_1(p)-G_2(p)-G_3(p)}{p^3-s^3-s^2-s-1}\right] \tag{1.3.25}$$

There we assume that the triple Laplace transform from exist for each term in the eight site (1.3.25).

An application to partial integral differential equation let us consider the following integral differential equation.

$$u_{ttt}-u_{xxx}-u_{xx}+u+\int_0^x\int_0^y\int_0^t g(x-\alpha,y-\beta,z-\gamma)d\alpha d\beta d\gamma=f(x,y,t) \tag{1.3.26}$$

With boundary condition

$$U(0,y,t)=f_1(t),\ u_x(0,y,t)=f_2(t),\ u_y(x,0,t)=f_3(t) \tag{1.3.27}$$

And initial condition

$$u(x,0,0)=g_1(x),\ u_y(x,0,t)=g_2(x),\ u_t(x,y,0)=g_3(x) \tag{1.3.28}$$

By taking Laplace transform for (1.3.27) and simple Laplace transform (1.3.27) and (1.3.28) we get

$$U(p,q,s) = \frac{\frac{p}{G_1(p)} + \frac{1}{G_2(q)} + \frac{1}{G_3(r)}}{p^3 - s^2 + 1 + G(p,s)} \tag{1.3.29}$$

By applying three inverse Laplace enter the we get the triple and Laplace transform solution and in the following form

$$U(p,q,s) = L_p^{-1} L_q^{-1} L_s^{-1} \frac{\frac{p}{G_1(p)} + \frac{1}{G_2(q)} + \frac{1}{G_3(r)}}{p^3 - s^2 + 1 + G(p,s)} \frac{\frac{p}{F_1(s)} + \frac{1}{F_2(s)} + \frac{1}{F_3(s)} + p(p,q,s)}{p^3 - s^3 - s^2 + 1 + G(p,s)} \tag{1.3.30}$$

We provide the three inverses Laplace transform for (1.3.29) and we set the solution (1.3.30).

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